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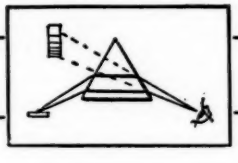

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Journal of the
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The Minimum Perceptible Colorimetric Purity as a Function of Dominant Wave-Length *

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National Bureau of Standards, Washington, D. C.
(Received March 23, 1938)

A 4° square, two-part photometric field, symmetrical about a vertical division and viewed through a pupil 3 mm in diameter, is illuminated in both parts by artificial sunlight at a constant brightness of about 3 or 4 millilamberts (retinal illumination, 70 to 90 photons) with a surrounding field of about 0.5 millilambert. Homogeneous light is added to one-half, and sunlight simultaneously subtracted so that the field remains matched in brightness. Two adjustments of the mixture are made: (1) the least purity perceptible with certainty (p_{max}), and (2) the greatest imperceptible purity (p_{min}). The purity of these mixtures is then measured, increased accuracy being obtained by measuring a known large multiple of the homogeneous brightness. Values of p_{max} and p_{min} have been obtained as a function of the wave-length of the homogeneous component; these values are reported in detail, and some discussion of their interpretation is given.

I. INTRODUCTION

THIS work was undertaken as a part of a program of research on determination and correlation of the characteristics of vision outlined by Priest[1] in 1921 when he listed as one of the functions to be determined, "Saturation sensibility—the least change in purity perceptible as a change in saturation." This work deals with saturation sensibility near zero purity.[2] It is therefore of particular interest in the colorimetry of illuminants closely resembling sunlight in color and in the colorimetry of nearly achromatic surface colors viewed under such illuminants; it bears importantly upon the question of tolerances in the specification of such colors and upon the determination of nearest color temperatures[3] by means of which such illuminants are customarily specified.

* In February, 1926, Irwin G. Priest and F. G. Brickwedde presented the results of their study of colorimetric purity before the Optical Society of America and an abstract, carrying the title given above, appeared in the J. Opt. Soc. Am. and Rev. Sci. Inst., **13**, 306 (1926). The work had not been prepared for publication at the time of Mr. Priest's death, and Dr. Brickwedde is now working in another field. Therefore, at the suggestion of the Director of the National Bureau of Standards, the writer has undertaken the preparation of this article.—Deane B. Judd.

* Publication Approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

† Research Associate at the National Bureau of Standards, representing the Munsell Research Laboratory, July 1925 to June 1926.

[1] Irwin G. Priest, "The Spectral Distribution of Energy Required to Evoke the Gray Sensation," Nat. Bur. Stand. Sci. Pap. **17**, 231 (1922); S417. See Appendix, p. 260.

[2] For a definition of colorimetric purity see Irwin G. Priest, "Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity and Brightness," J. Opt. Soc. Am. and Rev. Sci. Inst. **8**, 173 (1924); Irwin G. Priest, "The Computation of Colorimetric Purity," J. Opt. Soc. Am. and Rev. Sci. Inst. **9**, 503 (1924).

[3] R. Davis, "A Correlated Color Temperature for Illuminants," Nat. Bur. Stand. J. Research **7**, 659 (1931) RP365.

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II. Procedure

A. Adjustment of the mixture

A 4° square, two-part photometric field, symmetrical about a vertical division and viewed through a pupil 3 mm in diameter, was illuminated in both parts by Abbot-Priest sunlight[4] at a constant brightness of about 3 or 4 millilamberts (retinal illumination, 70 to 90 photons) with a surrounding field of about 0.5 millilambert.[5] Homogeneous light was added to the right half of the field and sunlight simultaneously subtracted so that the right half of the field remains matched in brightness with the unchanged left half. Two adjustments of the mixture in the right half-field were made. First the proportion (p) of homogeneous light in the mixture was increased until the observer perceived a definite chromaticity difference between the sunlight in the left half and the mixture in the right; this is called a setting of the least purity perceptible with certainty (p_{max}). The observer then decreased the proportion of homogeneous light in the mixture until the chromaticity difference just disappeared. This second adjustment is called the greatest imperceptible purity (p_{min}). The average between a setting of least perceptible difference and a setting of greatest imperceptible difference is thought to be more reliable an indication of sensibility than either one taken alone; it has sometimes been called the "doubtful difference."[6] Accordingly this average has also been computed and is referred to as (p_{mean}).

B. Photometric measurement of the components

After each adjustment (either of least perceptible or greatest imperceptible difference) the observer made a photometric measurement of the components of the mixture from which the purity (p_{max} or p_{min}) may be computed. One of two photometric methods was used.

[4] Irwin G. Priest, "A Precision Method for Producing Artificial Daylight," Phys. Rev. **11**, 502 (1918); J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 479 (1926). D. B. Judd, "Reduction of Data on Mixture of Color Stimuli," Nat. Bur. Stand. J. Research **4**, 525 (1930) RP163.

[5] Irwin G. Priest, "Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity and Brightness," J. Opt. Soc. Am. and Rev. Sci. Inst. **8**, 173 (1924).

[6] D. B. Judd, "Sensibility to Color-Temperature Change as a Function of Temperature," J. Opt. Soc. Am. **23**, 7 (1933).

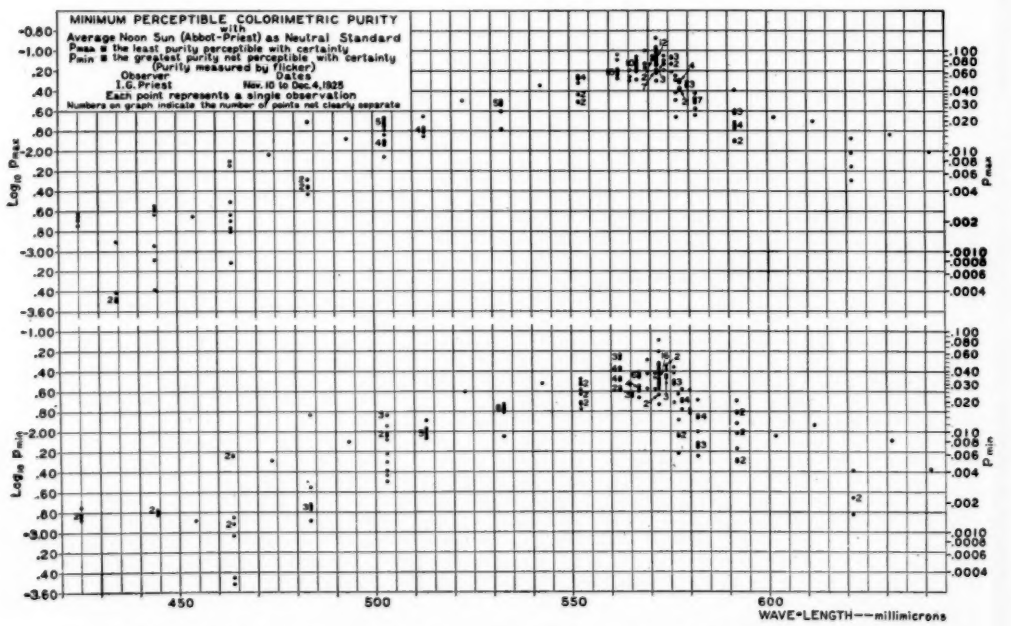

FIG. 1. Individual observations of minimum perceptible colorimetric purity (observer IGP), mixture analyzed by flicker method.

In the equality-of-brightness method, the nonselective filter in the homogeneous beam is removed. Since this filter was usually either of 5 or 10 percent transmission, the homogeneous component by this operation was increased either 20- or 10-fold. Then the observer decreased the sunlight component of the right half of the field until the brightness was again judged equal to that of the left half which was left unaltered throughout. The reduction in sunlight component was accomplished by increasing the distance of the lamp from a diffusing screen; it may therefore be expressed as a simple function of this distance (essentially the inverse-square law). This reduction is equal to a known multiple (usually about 19 or 9) of the homogeneous brightness, and thus permits the ratio of homogeneous brightness component to brightness of the mixture to be computed. This ratio is the colorimetric purity of the mixture.

The equality-of-brightness method is convenient but it is open to two criticisms: first, the field size (4° square) is larger than that to which the standard luminosity function recommended by the International Commission on Illumination applies,[7] and second, the observer has to make the photometric match with a considerable chromaticity difference (sometimes amounting to 20 least perceptible differences) between the two halves of the field. Accordingly, for about half of the work, the brightnesses of the components of the mixture were obtained by flicker photometry.

In the flicker method a 2° diaphragm was substituted for the rectangular 2° by 4° opening constituting the mixture field. The nonselective filter in the homogeneous beam was removed and the flicker disk interrupting the homogeneous beam with sunlight of brightness variable by an auxiliary photometer was set into rotation at a suitable speed. Adjustment of the auxiliary photometer for minimum flicker yields a measure of the brightness of this known multiple of the homogeneous component. From this brightness and the known constant brightness of the photometric field the purity of the mixture (p_{max} or p_{min}) was computed.

C. Precautions and auxiliary studies

The wave-length scale was calibrated by reference to the lines of the helium emission

[7] K. S. Gibson and E. P. T. Tyndall, "The Visibility of Radiant Energy," Nat. Bur. Stand. Sci. Pap. **19**, 131 (1923–24) S475. *Proceedings International Commission on Illumination*, 6th Meeting, Geneva (University Press, Cambridge, England, 1926), pp. 67, 232.

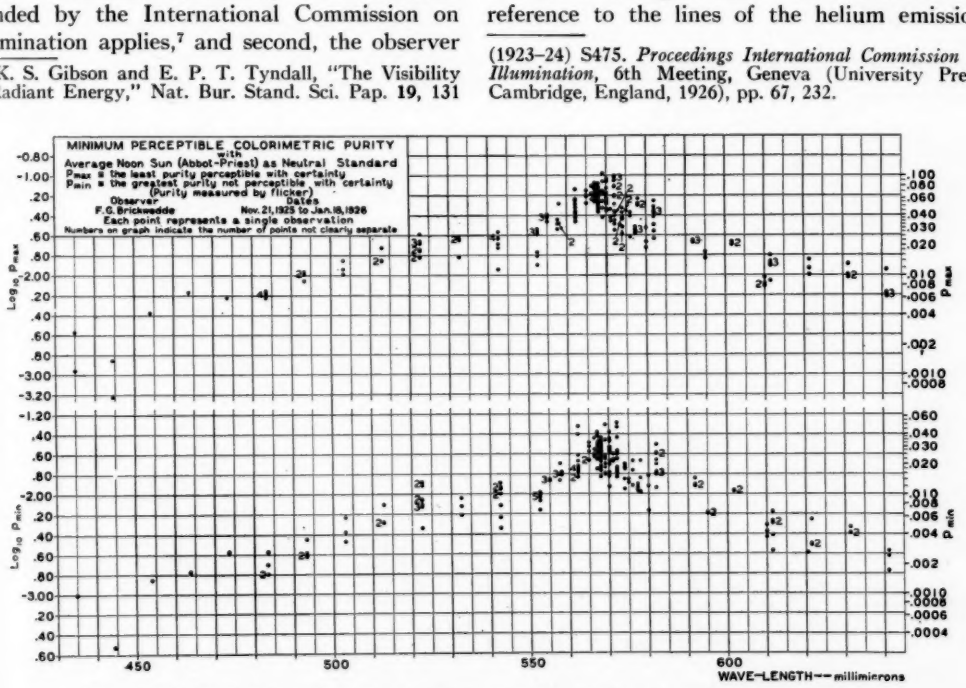

FIG. 2. Individual observations of minimum perceptible colorimetric purity (observer FGB), mixture analyzed by flicker method.

TABLE I.

Wave-Length in mμ	p_{mean}		Mean p_{mean}
	I.G.P.	F.G.B.	
405	0.00105	0.0013	0.00118
10	.00105	.0015	.00128
20	.0011	.00195	.00152
30	.00115	.0024	.00178
35	.0012	———	———
40	.0013	.00305	.00218
45	.0014	———	———
50	.0016	.00375	.00268
55	.0019	.00405	.00298
60	.00235	.0043	.00332
65	.00315	.0044	.00378
70	.0043	.0045	.0044
75	.00535	.0046	.00498
80	.00635	.0047	.00552
85	.0074	.0050	.0062
90	.0086	.00565	.00712
95	.00995	.0065	.00822
500	.0115	.00755	.00952
05	.0131	.00875	.01092
10	.0149	.0102	.01255
15	.0169	.0118	.01435
20	.0190	.0132	.0161
25	.0214	.0143	.01785
30	.0238	.0148	.0193
35	.0264	.01495	.02068
40	.0286	.01505	.02182
45	.0300	.0152	.0226
50	.0308	.0166	.0237
52.5	.0312	.0189	.02505
55	.0320	.0225	.02725
57.5	.0340	.0272	.0306
60	.0387	.0322	.03545
62.5	.0454	.0374	.0414
65	.0516	.0422	.0469
67.5	.0564	.0447	.05055
568	———	.0447	———
70	.0594	.0432	.0513
71.5	———	.0405	———
72.1	.0604	———	———
72.5	.0603	.0364	.04835
73.5	.0595	.0306	.04505
75	.0548	.0262	.0405
76	.0452	———	———
77.5	.0340	.0229	.02845
78.5	.0289	———	———
80	.0258	.0208	.0233
82.5	.0226	———	———
85	.0206	.0176	.0191
90	.0176	.0150	.0163
600	.0137	.0114	.01255
10	.0111	.0087	.0099
20	.0090	.0073	.00815
30	.00745	.00685	.00715
40	.0062	.00655	.00638
50	.00545	.00635	.0059
60	.0051	.0062	.00565
70	.0049	.006125	.00551
80	.00485	.00605	.00545

spectrum. Stray light in the spectrum was measured and stray-light filters used in the extremes of the spectrum over the whole region within which stray light would introduce appreciable errors. This is particularly important for the short wave region of the spectrum.

A special check was made (observer IGP) of the slit width errors. These errors can be shown by computation to be negligible in comparison to the inherent uncertainty of liminal settings even for the rather large slit widths used (10 mμ). The experimental check corroborated the computations.

A special study was made (observer FGB) of the variation of p_{max} and p_{min} taken on different days over a period of six weeks. It was found that there was no regular variation from day to day, but the mean for a given day varied on the average from the mean on another day by more than would be expected from the theory of errors. This is probably to be ascribed to a change in criterion of what constitutes a "just perceptible" and a "just imperceptible" difference; such variations are common, and indeed, characteristic of liminal settings.[8] The conclusions from this study were used in a special determination of the wave-length of the maximum value of p_{mean} (observer, FGB). This determination was carried out by making one setting only of p_{max} and p_{min} at one wave-length (565.0 mμ) somewhat lower than that which preliminary determination indicated for the maximum, then a single determination approximately at the wave-length of the maximum (567.5 mμ) and finally a single determination at a wave-length (570.0 mμ) slightly above that for the maximum. After a rest period, the observer repeated the series starting with the higher wave-length. In this way, changes in criteria were avoided (long time fluctuations), and practice and fatigue effects (short time fluctuations) reduced. From this work it was concluded that the wave-length at which p_{mean} is a maximum is for this observer (FGB, left eye) 568.0 ±1.0 mμ.

[8] See, for example, D. B. Judd, "Precision of Color Temperature Measurements under Various Observing Conditions; a New Color Comparator for Incandescent Lamps," Nat. Bur. Stand. J. Research **5**, 1164 (1930); RP252.

III. Results

Figure 1 shows on a logarithmic scale as a function of wave-length the result of every separate determination taken by IGP by the flicker method. Fig. 2 similarly shows those taken by FGB.

Table I gives the average values of p_{mean} for each observer. These averages were obtained by plotting points representing the averages of the groups of points shown on Figs. 1 and 2 on a logarithmic scale together with similar points derived from data obtained with the equality-of-brightness method. Smooth curves were drawn among these points, greater weight being given to the flicker determinations. The flicker method is not open to the two criticisms of the equality-of-brightness method discussed in the previous section, and it has the further advantage of greater precision in the measurement of the smaller homogeneous brightnesses. This greater precision is particularly marked for the extremes of the spectrum (400 to 450 mμ, 630 to 680 mμ); in these regions it sometimes happened that the equality-of-brightness method even gave negative values for p_{min} in spite of the nature of the experiment which insures that some amount of homogeneous component greater than zero must be added. It will be understood, however, that even when increased by 10- or 20-fold these very small homogeneous brightnesses added to the sunlight component may fall below the differential limen for equality-of-brightness settings. It may therefore be concluded that the addition to sunlight of small amounts of light from the extremes of the spectrum are detectable as chromaticity differences long before they become detectable as brightness differences.

The only regular difference observed between the flicker and equality-of-brightness methods is that for the long wave half of the spectrum the flicker method gave generally higher results. This is to be expected from the smaller field size, and since greater interest attaches to photometric results in accord with this smaller field size, the general course of the flicker determinations was followed closely in drawing the curve. The greater weight attached to the flicker determinations was therefore of two kinds: that attached to more representative photometric observing conditions and that to determinations of higher precision.

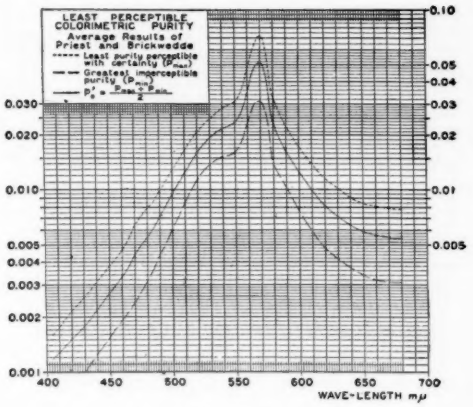

Fig. 3. Minimum perceptible colorimetric purity, average results of Priest and Brickwedde.

The values of the first two columns in Table I were computed from values of p_{max} and p_{min} read from these adopted curves at every five millimicrons. The values in the last column of Table I are the arithmetical means of the values in the first two columns.

Figure 3 shows (solid curve) a plot on a logarithmic scale of a smooth curve drawn through the values given in the last column of Table I. It also shows curves of p_{max} (dotted curve, arithmetical mean of p_{max} for IGP and p_{max} for FGB) and p_{min} (dashed curve, arithmetical mean of p_{min} for IGP and p_{min} for FGB).

IV. Discussion

A. Comparison with other work

When these data were obtained there was only one investigation with which to compare them; this was the pioneer work of Jones and Lowry[9] on saturation scales extending from zero purity to the spectrum (unit purity) for eight different wave-lengths. Although subsequent repetition of this work has in the main corroborated it, the variation with dominant wave-length of the first saturation step from zero purity found by Jones and Lowry incidental to their main purpose is much less than that

[9] L. A. Jones and E. M. Lowry, "Retinal Sensibility to Saturation Differences," J. Opt. Soc. Am. and Rev. Sci. Inst. 13, 25 (1926).

reported here. For example, Jones and Lowry give 0.035 as the size of the first purity step for 575 mμ and 0.025 for 440 mμ; this is a variation by a factor of about 1.4, while from Table I the corresponding variation is by a factor of about 18.

Since 1926, however, p_{max} has been redetermined by no less than three independent investigations, those by Purdy,[10] by Martin, Warburton and Morgan,[11] and by Wright and Pitt.[12] Purdy investigated the change in p_{max} with field brightness and therefore extended the scope of his work over that of the present results in an important particular; but he used stimuli of relatively few wave-lengths (8) and over the range 400 to 660 mμ only. Martin, Warburton and Morgan supply data for three observers both for light adaptation and dark adaptation; their investigation refers to 21 wave-lengths distributed over the range 440 to 680 mμ. Wright and Pitt obtained data at 19 wave-lengths distributed uniformly over the range 470 to 650 mμ. In each of these investigations the results completely corroborate within experimental error those previously obtained by Priest and Brickwedde with exceptions arising only from the fact that the Priest-Brickwedde results cover a greater wave-length range more thoroughly than any other investigation.

There can therefore be no doubt remaining as to the essential correctness of the data presented here completely for the first time.[13]

B. Interpretation

Through the courtesy of Mr. Priest these data were in advance of publication placed at the disposal of Dr. Selig Hecht and the writer. By an extension of the Young three-components theory of vision Hecht has shown that these data are consistent with sensibility to wave-length change in the spectrum and roughly consistent with the mixture data of the OSA "excitation" curves used for many years as a definition of normal vision.[14] These data have formed an important basis for Hecht's further theoretical papers in which he has shown that they may be completely harmonized with the OSA curves.[15] Judd has shown by an empiric relation that these data are consistent with many other chromaticity-sensibility functions and are approximately consistent both with the mixture data and coordinate system of the OSA "excitation" curves,[16] and he has also shown by a special transformation of coordinates that these data are in precise agreement with the mixture data of the standard observer adopted in 1931 by the International Commission on Illumination.[17]

From appearance in summarized form in these publications the Priest-Brickwedde data have continued to spread[18] and in this way have already exerted a considerable influence on modern developments in visual theory.

It is probably worth noting that Priest for some time held the view that p_{mean} should duplicate very closely the luminosity function. This caused him to remark[19] "Least perceptible

[10] D. McL. Purdy, "On the Saturations and Chromatic Thresholds of the Spectral Colours," Brit. J. Psych. (Gen. Sec.) **21**, 283 (1931).

[11] L. C. Martin, F. L. Warburton and W. J. Morgan, "Determination of the Sensitiveness of the Eye to Differences in the Saturation of Colours," Medical Research Council, Reports of the Committee upon the Physiology of Vision, XIII, Special Report Series, No. 188, London, 1933.

[12] W. D. Wright and F. H. G. Pitt, "The Saturation-Discrimination of Two Trichromats," Proc. Phys. Soc. **49**, 329 (1937).

[13] Martin, Warburton and Morgan (see reference 11) give a discussion of possible sources of unreliability of the Jones-Lowry data. See, also, D. B. Judd, "Chromaticity Sensibility to Stimulus Differences," J. Opt. Soc. Am. **22**, 95 (1932).

[14] Selig Hecht, "The Development of Thomas Young's Theory of Color Vision," J. Opt. Soc. Am. **20**, 231 (1930). The tentative theoretical curves in this paper do not completely harmonize mixture data for the normal eye with the Priest-Brickwedde data; consult D. B. Judd, "The Mixture Data Embodied in the Tentative Curves of Hecht's Theory of Vision," J. Opt. Soc. Am. **20**, 647 (1930); this deviation has been taken care of in Hecht's subsequent formulation of Young's idea.

[15] Selig Hecht, "The Interrelation of Various Aspects of Color Vision," J. Opt. Soc. Am. **21**, 615 (1931); "A Quantitative Formulation of Colour-Vision," Report of a Joint Discussion of Vision, The Physical and Optical Societies, June 1932; "The Retinal Processes Concerned with Visual Acuity and Color Vision," Bulletin No. 4, Howe Laboratory of Ophthalmology, Harvard Medical School, Harvard University Press, Cambridge, August 1931.

[16] D. B. Judd, "Chromaticity Sensibility to Stimulus Differences" J. Opt. Soc. Am. **22**, 72 (1932).

[17] D. B. Judd, "A Maxwell Triangle Yielding Uniform Chromaticity Scales," Nat. Bur. Stand. J. Research **14**, 41 (1935) RP756; also J. Opt. Soc. Am. **25**, 24 (1935).

[18] See, for example, J. F. Schouten, "Grundlagen einer quantitativen Vierfarbentheorie. I," Proc. Koninklijke Akad. v. Wetenschappen, Amsterdam, **38**, No. 6 (1935). Schouten, like Hecht, has shown an approximate correspondence between mixture data for the normal eye and the Priest-Brickwedde data by resort to a certain form of visual theory.

[19] I. G. Priest and F. G. Brickwedde, "The Minimum Perceptible Colorimetric Purity as a Function of Dominant Wave-Length with Sunlight as Neutral Standard," J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 306 (1926).

purity is largely dependent upon visibility; the relative amounts of *energy* required to introduce a hue difference show comparatively small variation with wave-length," and to view with suspicion the presence (see Fig. 3) of the marked maximum quite definitely removed (570 mμ) from the wave-length of maximum spectral luminosity (555 mμ). He was inclined to the belief that if a truly achromatic stimulus were to be substituted for the rather greenish-yellow artificial sunlight then this pronounced maximum at 570 mμ would no longer be found. This belief led him to remark, "Further extensive investigation of least perceptible purity should be prefaced by a definite adoption of a neutral standard."

The present view of this maximum at 570 mμ is that it indicates sunlight to be more confusible in chromaticity with 570 mμ than with any other part of the spectrum. On the basis of this conclusion a new coordinate system was derived[17] which has not only been successful in coordinating chromaticity sensibility data of various sorts but has also proven to be useful in practical specification of color tolerances and in the estimation for practical purposes of nearest chromaticity from a chromaticity series.[20] The success of these colorimetric procedures can largely be traced to the Priest-Brickwedde data.

[20] D. B. Judd, "A Method for Determining Whiteness of Paper," Paper Trade J., Tech. Sec. **100**, 266 (1935); **103**, 154 (1936); also Tech. Assoc. Papers, Series 18, 392 (1935); Series 19, 359 (1936). H. J. McNicholas, "Color and Spectral Transmittance of Vegetable Oils," Nat. Bur. Stand. J. Research **15**, 99 (1935) RP815; also Oil and Soap **12**, 167 (1935). D. B. Judd, "Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 ICI Colorimetric Coordinate System," Nat. Bur. Stand. J. Research **17**, 771 (1936) RP944; also J. Opt. Soc. Soc. Am. **26**, 421 (1936). H. J. McNicholas, "Selection of Colors for Signal Lights," Nat. Bur. Stand. J. Research **17**, 955 (1936) RP956. D. B. Judd, "Surface Color," J. Opt. Soc. Am. **25**, 44 (1935). F. C. Breckenridge and W. R. Schaub, "Rectangular Uniform-Chromaticity-Scale Coordinates," J. Opt. Soc. Am. **27**, 226 (1937). D. L. MacAdam, "Projective Transformations of I.C.I. Color Specifications," J. Opt. Soc. Am. **27**, 294 (1937).

Suggestions Concerning Infra-Red Prism Spectroscopy

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(Received February 14, 1938)

A discussion is given of the present existing limits of usefulness of the various prisms commonly employed in the infra-red. A new method for extending some of these limits is proposed. This method, based upon the disappearance of newly found secondary absorption maxima at low temperatures, would require that prisms of the alkali halides be used while cooled with liquid air. A possible spectrometer for this purpose is presented. Transmission curves for LiF crystals are given which show that a prism of this material would be useful to about 6μ.

WITH the rather recent introduction of prisms of synthetically grown crystals of KBr and KI, the range of prism spectroscopy has been extended into the infra-red to about 31μ, or 320 cm^{-1}. If one wishes to make measurements beyond this limit, he is forced to make use of a grating spectrometer employing either an echelette or a wire grating. The increased difficulties of grating spectroscopy, chief among which is the problem of elimination of overlapping orders of shorter wave-lengths, have led many infra-red investigators to search for some means of extending the range of the prism. One obvious method would be to produce large single crystals of the heavier alkali halides, such, for example, as rubidium or cesium bromide or iodide. Thallium halides also crystallize in the cubic system and so might conceivably be useful as prisms, in which case Tl I would be the most important. From Fig. 1, which is reproduced from an earlier publication,[1] one can see at a glance the infra-red characteristics of the alkali halides, namely the wave-lengths of the characteristic frequencies ν_0, and the reststrahlen. Table I shows the practical limits of the usefulness of prisms, L, made from the various materials as compared with their characteristic frequencies, ν_0. Using NaCl, KCl, KBr, and KI, for which exact transmission measurements on relatively thick samples exist, one can see that, with the exception of LiF, the ratio L/ν_0 is roughly constant. If we take this constant to be 3.3, we can, by using the known values for ν_0, make reasonable predictions as to the limits for the heavier crystals. From such predictions little hope may be derived for extending prism spectroscopy far beyond 50μ or 200 cm^{-1}.

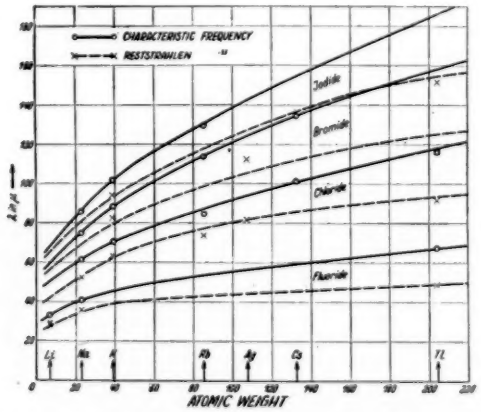

FIG. 1. Infra-red characteristics of alkali halides, wave-lengths in μ plotted against atomic weights.

As a result of various studies of the NaCl crystal, it occurred to the writer that there was an entirely different approach to this question, based upon the use of a prism at low temperatures, which might lead to very useful results. In order to test this idea, an experiment was undertaken and an apparatus constructed at Princeton University. Unfortunately, it has been impossible to carry the experiments through to completion. The present discussion of the proposed experiment is offered at this time in the sincere hope that some other investigator may see fit to carry on the work.

Reason for Practical Limit

The use of the alkali halides for prisms in infra-red spectroscopy is based upon the fact that, since they are cubic crystals, they each possess, as is shown in Born's theory of crystals, only one

[1] R. Bowling Barnes, "Die ultraroten Eigenfrequenzen der Alkalihalogenidkristalle," Zeits. f. Physik **75**, 731 (1932).

major absorption band and this in the far infra-red. Fig. 2 shows the changes which occur in the varous optical properties of the crystal in the neighborhood of the characteristic frequency ν_0. From this figure, one sees that (a) the transmission T of a thin plate of NaCl is a minimum at ν_0, (b) the reflection R is a maximum at a slightly higher frequency than ν_0, (c) the refractive index n shows "anomalous dispersion" around ν_0, (d) the absorption coefficient nK shows a maximum at ν_0, and (e) the reflection curve R shows definite evidence of a "secondary maximum" on the high frequency side of ν_0. Similar results occur in the vicinity of other crystal absorptions.

Looking at curve T, one might wonder why NaCl, if it has its ν_0 at 61μ, should become opaque at 16μ. The answer, as has been shown in a series of papers,[2] lies in the fact that these cubic crystals all possess in addition to ν_0 many secondary maxima of absorption. The great majority, and by far the stronger, of these lie on the high frequency side of ν_0 in the neighborhood of $2\nu_0$ and $3\nu_0$. As a matter of fact, Seitz [3] has calculated on the basis of group theory that some 260 of these maxima should exist at room temperature. The same calculations show that, if the crystal is reduced to a temperature below the characteristic temperature (say -180°C for NaCl) only 60 of the 260 frequencies should persist. This would obviously result in a marked increase of transparency in the neighborhood of $2\nu_0$ and $3\nu_0$, making it conceivable that a prism of NaCl cooled to this temperature might be used further out into the infra-red than its present limit of 16μ.

TABLE I.

Salt	ν_0 (cm^{-1})	L (cm^{-1})	L/ν_0
LiF	306	1540 (10μ) obs.	5.0
NaCl	164	588 (17μ) "	3.6
KCl	142	455 (22μ) "	3.2
KBr	113	356 (28μ) "	3.16
KI	98	320 (31μ) "	3.25
RbBr	88	290 (34μ) calc.	
RbI	77	254 (39μ) "	
TlI	67	222 (45μ) "	

[2] M. Czerny, Zeits. f. Physik **65**, 600 (1930); R. Bowling Barnes and M. Czerny, Zeits. f. Physik **72**, 447 (1931); R. Bowling Barnes, Zeits. f. Physik **75**, 723 (1932); M. Born and M. Blackman, Zeits. f. Physik **82**, 551 (1933); M. Blackman, Zeits. f. Physik **86**, 421 (1933).

[3] R. Bowling Barnes, R. Robert Brattain and Frederick Seitz, Phys. Rev. **48**, 582 (1935).

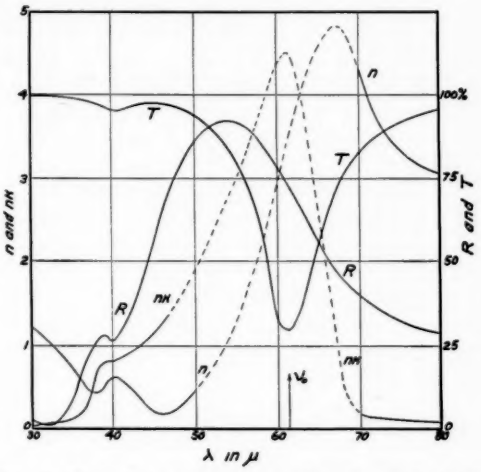

FIG. 2. Changes in optical properties of crystals in the neighborhood of their characteristic frequencies.

Should this become practical, one might make the common NaCl prism do the work of the much rarer prisms of KBr and KI. Since the prism could be used closer to its region of anomalous dispersion, its useful dispersion would be much higher, and it would immediately take on a new and added importance. If such an experiment should prove successful for NaCl, it would undoubtedly enable one, using a KI (or CsI) prism, to make measurements with a prism spectrometer in a range of the spectrum hitherto accessible only with a grating.

EXPERIMENTAL BASIS FOR PROPOSAL

Examination of the literature shows that there is also some experimental evidence which shows that by lowering the temperature of a crystal, the transmission at wave-lengths shorter than the characteristic absorption may be expected to increase greatly. Long before secondary maxima were discovered, Rubens and Hertz[4] measured the temperature coefficients of several crystals at various wave-lengths. Their results showed that for wave-lengths far from a characteristic absorption band the slope of the curve, percentage of incident energy which is absorbed as ordinates against the temperature as abscissae, was such as to "allow complete transparency to be expected

[4] Von H. Rubens and G. Hertz, Sitzungsberichte der Koniglich Preussischen **14**, 256 (1912).

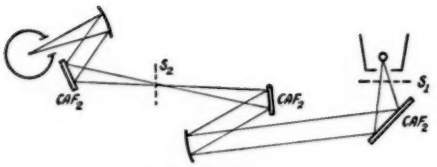

FIG. 3. CaF_2 reststrahlen spectrometer.

at absolute zero." For wave-lengths close to an absorption band, on the other hand, the curves flattened out rapidly as the temperatures were lowered, and showed that, even at extremely low temperatures, quite an appreciable absorption still remained. Surely this wave-length dependence of the temperature coefficients indicates the disappearance of the secondary absorption maxima with decreasing temperatures.

These temperature coefficients are surprisingly large. Rubens and Hertz found, for example, that for NaCl at 23μ the value of the extinction coefficient K decreased from 1.16×10^3 at 100°C to 0.13×10^3 at −186°C. For quartz at 7μ (close to an absorption), K only dropped from 7.45×10^3 at 100°C to 4.93×10^3 at −186°C. One must expect, therefore, to find that at a wave-length removed from the characteristic absorption, the percentage transmission of a given sample of a crystal will vary rapidly with changes in temperature. Czerny and Barnes[5] found that the transmission of a certain plate of NaCl varied some three or four percent for only four or five degrees difference in temperature. The plate of NaCl placed at S_1 of a fluorite reststrahlen spectrometer (Fig. 3) transmitted about 8 percent less than when placed at S_2. Careful determinations showed a temperature difference of 10°C. After this difference was eliminated by allowing an electric fan to blow upon the sample when at S_1, the two percentage transmissions were identical.

PROPOSED EXPERIMENT

While discussing the theory of secondary maxima with F. Seitz,[3] the author decided to construct a new vacuum spectrometer in which the prism could be cooled to liquid air temperatures. Immediately several important questions regarding the design of such an instrument arose, the

[5] R. Bowling Barnes, unpublished results.

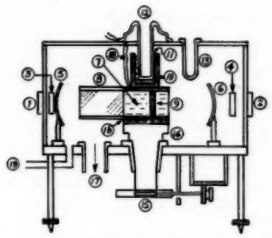

FIG. 4. Side view of spectrometer.

tentative solutions to which are shown in the drawings of the apparatus as constructed. A simple spectrometer of the Wadsworth type was built and placed in a strong brass case which was properly braced to withstand evacuation. Fig. 4 shows a side view of this spectrometer. In this, (1) and (2) are KBr plates, (3) and (4) are slits, (5) and (6) concave mirrors, (7) is a NaCl prism, (8) is a plane Wadsworth mirror, (9) a copper prism mount, (10) a copper cup, (11) powdered copper, (12) and (13) are Dewar flasks, (14) a vacuum-tight cone-shaped joint for rotating the prism, (15) a layer of felt for insulation, (16) a tangent screw reading device, (17) lead to vacuum, and (18) thermocouple pump.

The cooling arrangement and prism mount are shown again in Fig. 5. As the prism is rotated, the milled top plate of the prism mount rotates under the milled bottom of the copper cup. The powdered copper allows for the difference in expansion between the glass and the copper. The felt pad and the leather washers insulate the cold prism mount from the rest of the brass case, enabling the KBr windows to remain at room

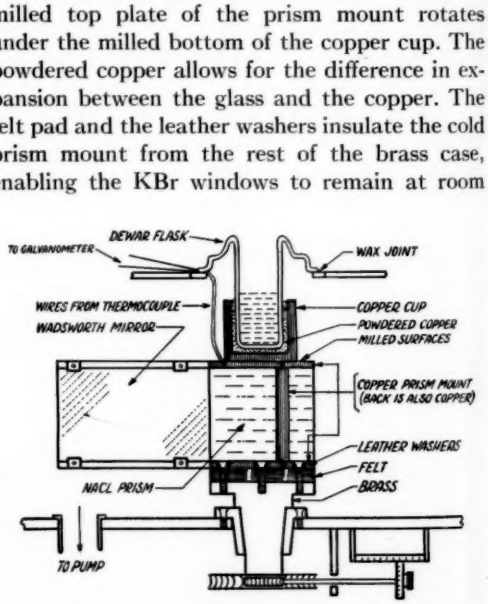

FIG. 5. Prism mount and cooling device.

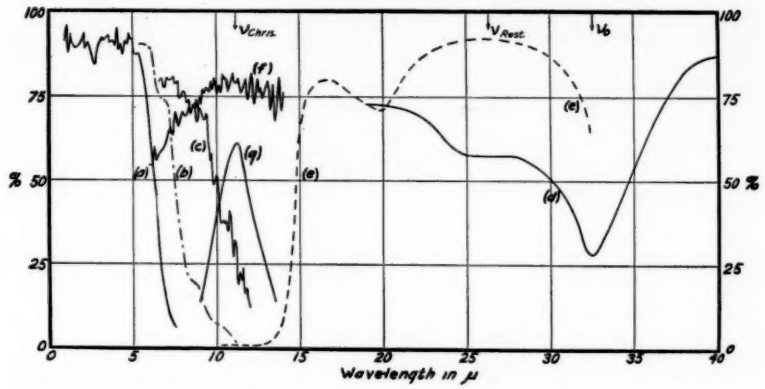

FIG. 6. Infra-red story of LiF crystals. (a) and (b), thin cleavages of LiF; (c) and (d), evaporated films of LiF; (e), reflection from a LiF plate; (f), layer of finely divided LiF particles; (g) Christiansen peak obtained by transmission through a layer of coarse LiF particles.

temperature. The cut Dewar flask is waxed to the top of the brass case. A Knudsen gauge was connected, (19) in Fig. 4, to the case in order to measure the true pressure.

It was intended that in the course of a series of measurements a definite routine should be followed in order to protect the prism surfaces. The case would be evacuated and the auxiliary Dewar flask, (13) in Fig. 4, then be filled with liquid air in order to condense any vapors which might still be present. After a few minutes, the cooling of the prism could be begun. Care would have to be taken, of course, not to cool the prism too rapidly. It might even be necessary to cool it down in several stages, making use of several different cooling solutions. The prism, once cooled, would then have to be calibrated, for its dispersion is a function of the temperature, which could be measured by means of the thermocouple, (18) in Fig. 4.

Undoubtedly, many minor difficulties will accompany such an experiment, but nevertheless the proposal as such seems reasonable. It remains to be seen whether it can be made practicable.

TRANSMISSION OF LiF CRYSTALS

The samples used were obtained from C. D. Stockbarger, of M.I.T., and crystallized by him from the very purest of raw material. Fig. 6 shows the complete infrared story of LiF crystals as determined to date.[6] As is shown, a plate 2.0 mm thick is 75 percent transparent at 7μ and opaque at 10μ. Thus, a prism made of LiF could be used to about 6.0 or 7.0μ only. The secondary maxima referred to above show up as fine transmission minima which are responsible for the heavy absorption in the short wave-lengths, the characteristic absorption being at 32.6μ. The secondary maxima also show up as reflection maxima in the reflection curve taken from a paper of Korth.[7] The Christiansen peak[8] is shown at 11.2μ, the wave-length at which the refractive index of LiF equals *I*, the refractive index of air.

[6] R. Bowling Barnes and Lyman G. Bonner, Phys. Rev. **49**, 648 (1936).
[7] K. Korth, Nachr. d. Ges. d. Wiss. zu Gott. Math-Phys. Kl. 576 (1932).
[8] R. Bowling Barnes and Lyman G. Bonner, Phys. Rev. **49**, 732 (1936).

Motion-Picture Polarigraph

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Berkeley, California

(Received March 21, 1938)

An instrument has been developed for replacing visual measurements of polarization, transmission, etc. by permanent records. A set-up is presented for determining the polarization of sky light.

I. INTRODUCTION

POLARIZATION measurements in the laboratory and in the field have been carried out, as a rule, visually. It often seems desirable, however, to secure permanent records of the phenomena observed. Several authors tried to replace the human eye by a photoelectric cell: Wright[1] measured the polarization of the moon, and Sekera[2] that of the sky in the zenith. Well-known difficulties in this type of work are the change in sensitivity of the cell and the lag in the readings of the galvanometer. These difficulties may be avoided by using photographic cameras equipped with Nicol or Wollaston prisms, or a combination of a Savart polariscope with a photographic camera. Records are taken on photographic plates, and the polarization of the sky,[3] of the moon,[1] or of the solar corona[4] is obtained at a given time. This paper deals with a motion-picture polarigraph used for (a) recording changes of polarization of a light source, for instance in polarimetric work, and (b) surveying extended light sources, for instance, the sky.

II. MOTION-PICTURE POLARIGRAPH

A Martens polarization photometer[5] was rebuilt. Fig. 1 shows the optical arrangement. The beam of light enters the diaphragm *a*, passes through lens *b*, Wollaston prism *c*, and biprism *d*. A fine thread *h* and a pointer *g* are connected to sleeve *f* which supports also the Nicol prism *e*. Parts *f*, *e*, *g*, and *h* may be rotated together, whereas *b*, *c*, and *d* are stationary. Lens *i* is focused on the dividing line of *d*. The photometric image is obtained on the 16 mm film *k* of a motion-picture camera (make Bell and Howell, Automatic Ciné Camera, with adjustable speed and spring motor). The photometric image may be viewed on the ground glass *m* after inserting prism *l* into the optic axis. The ocular *n* may be used for magnifying the image.

The sleeve for the Nicol prism is connected, by means of gears and transmission, to the motor of the motion-picture camera, Fig. 2. The

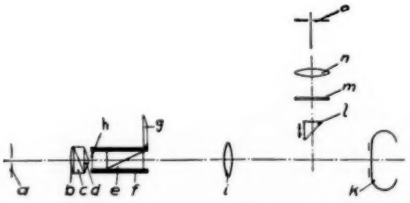

FIG. 1. Optical arrangement of motion-picture polarigraph. *a*, diaphragm; *b*, lens; *c*, Wollaston prism; *d*, biprism; *e*, Nicol prism; *f*, sleeve; *g*, index; *h*, thread; *i*, lens; *k*, motion-picture film; *l*, prism; *m*, ground glass; *n*, lens; *o*, exit pupil.

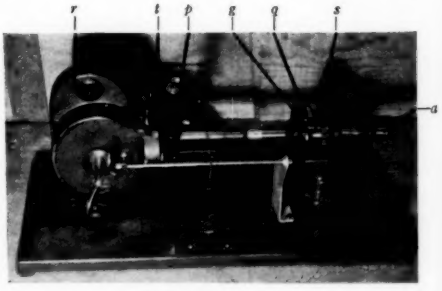

FIG. 2. Motion-picture polarigraph. *a*, diaphragm; *g*, index for Nicol prism and thread; *p*, exit pupil cover; *q*, fixed pointer; *r*, motion-picture camera; *s*, Martens polarization photometer; *t*, camera release.

[1] F. E. Wright, Sci. Monthly **40**, 101–115 (1935).

[2] Z. Sekera, Gerland's Beiträge zur Geophysik **39**, 285–299 (1933); **44**, 157–175 (1935).

[3] H. F. Newall, Monthly Notices Royal Astron. Soc. **66**, 475–481 (1906); A. Wegener, Sitzungsberichte Ges. Förd. Naturw. Marburg **1914**, 7–18 (2); W. Voss, Himmelswelt **43**, 112–113 (1933).

[4] Willi M. Cohn, "Polarization of the solar corona in the total eclipses of 1932 and 1934," Astrophys. J. **87**, 284–334 (1938).

[5] F. F. Martens, Physik. Zeits. **1**, 299–303 (1899/1900); Photograph. Korr. **38**, 528–531 (1901); J. J. Tichanowsky, Physik. Zeits. **25**, 482–484 (1924).

FIG. 3. Motion-picture film HP-12. *h'*, picture of thread *h*; *I*, *II*, photometric fields.

speed of rotation of the Nicol prism is adjusted for obtaining 35 pictures on the film for one revolution of the Nicol prism. Each picture on the film shows the photometric field, together with an image of thread *h*, indicating the position of the Nicol at the time of each exposure. The entire photometer may be further rotated in the optic axis (a coupling being provided in the transmission).

III. MEASUREMENT OF POLARIZATION

Index *g* is fixed in the zero position. The entire photometer is rotated until the two halves of the photometric field in *m* match. The position of the instrument is read by means of pointer *q*, Fig. 2, and the plane of polarization determined. The entire photometer is rotated by 45 degrees (to pointer *u*, Fig. 6), setting the principal plane of the Nicol prism at 45 degrees to the planes of vibration of the transmitted beams. It should be mentioned that, in many cases, no determination of the plane of polarization, and, in other cases, only casual determinations of the plane of polarization are necessary.

The polarization is obtained as follows: Prism *l* is moved into the position of Fig. 1 closing simultaneously the exit pupil *o*, and *g* released. The camera motor is started and the exposures are taken. A rotation of the Nicol prism by 90° (9 exposures) would be sufficient for obtaining the polarization; it is preferable, however, to rotate the Nicol by 360°, in order to check the measurement.

The next set of exposures is taken with the light source under different conditions, or after inserting a polarizing substance, or after directing the instrument toward another light source. Fig. 3 presents part of a record of a polarimetric measurement. The exposures show opposite changes of the density of the two halves of the photometric field as a function of the position of the Nicol indicated by the position of *h'*. (The record was taken on Eastman positive film, which is rather inexpensive and slow, but gives a high contrast. The film was developed by means of a Stineman spiral-shaped reel[6] in Eastman D-19 developer.)

The polarization is obtained from the azimuth of the Nicol prism at which both photometric fields match. The measurement of the records may be carried out by any suitable densitometer. The Martens polarization photometer itself may be used for this purpose, using two incident beams from the same lamp, one of which serves as a comparison beam, whereas the other one passes through the film to be measured. Fig. 4 shows such a set-up.

Figure 5 presents some results of the measurement of film HP-12, Fig. 3. Curves of the photographic density of the photometric fields *I* and *II* are derived as a function of the azimuth of the Nicol prism. It will be seen that the curves intersect at position angle 269.2°. The polarization amounts to 0.027_9. The transmission of a sample is obtained correspondingly. The graphic determination of the intersection of the density curves seems justified because the

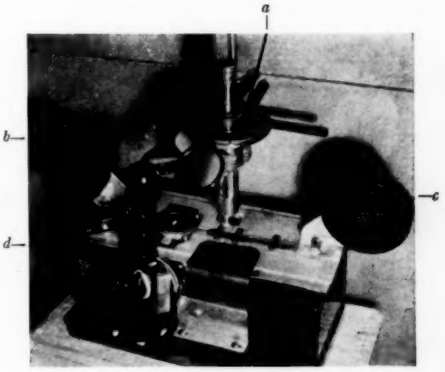

FIG. 4. Densitometer set-up for motion-picture film. *a*, Martens polarization photometer; *b*, *c*, film reels; *d*, circle and vernier with double cross hairs for measuring azimuth of *h'*.

[6] J. I. Crabtree, Trans. Soc. Motion Pict. Eng. 163–211 (16) (1923).

density of the images on the film lies, near the point of intersection, within the straight part of the characteristic (blackening) curve of the film. The results of the measurements are reproducible. The accuracy of the measurements is, at least, equal to that of visual determinations; it depends here on the accuracy which is obtained in reading the position of h' in Fig. 3, whereas it depends, in visual work, primarily on the accuracy of matching the halves of the photometric field. The accuracy may be increased here by an attachment as used in saccharimetric work. The probable error of the measurement of film HP-12 amounts to ± 0.18 degrees.

IV. Applications of the Motion-Picture Polarigraph

The instrument was developed originally for the determination of the *polarization of sky light*, after we found that short period changes occur in the polarization of sky light during an eclipse of the sun.[7] These changes do not extend, however, to the immediate vicinity of the sun.[8] It seems desirable to continue these investigations over the entire sky, both with and without an eclipse, and an attempt should be made to discover any further rapid changes of polarization and the reasons for their occurrence. The arrangement used for sky light work is shown in Fig. 6. A survey of the sky may be carried out in a few minutes. We expect to report some results later.

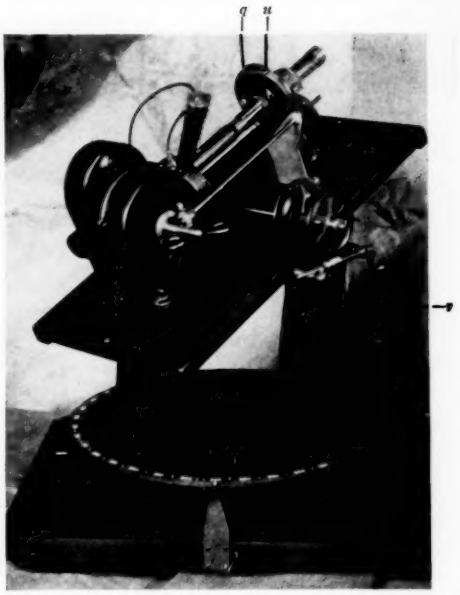

Fig. 6. Motion-picture polarigraph assembly for sky light measurements. q, u, fixed pointers; v, mounting for moving the instrument in declination and azimuth.

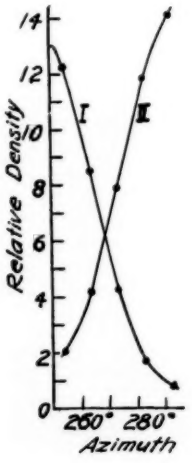

Fig. 5. Results of measuring part of film HP-12, Fig. 3.

It should be mentioned here that the polarization of sky light seems to be a very sensitive supplement to routine measurements of radiation from sun and sky and of the turbidity of synoptic air masses. It seems desirable to find out if, for instance, the approach of typhoons may be predicted from polarization measurements, since changes in the polarization of sky light seem to occur a long time before any clouds, etc., appear.[9] Another interesting application seems a study of changes of the "neutral points" in sky light which may also help in weather forecasting, see for instance the visual determinations by Jensen.[10]

Further applications of the instrument are obvious for polarimetric work in the laboratory

[7] W. M. Cohn, Phys. Rev. **45**, 848 (1934); Nature **134**, 99 (1934).

[8] W. M. Cohn, "Polarization of skylight close to the sun," to appear in Gerland's Beiträge zur Geophysik.

[9] F. Busch and C. Jensen, *Jahrbuch Hamburg. Wiss. Anstalt.* Vol. 28 (1910), 5. Beiheft; N. N. Kalitin, Meteorolog. Zeits. **41**, 9–15 (1924); W. Smosarski, Gerland's Beiträge zur Geophysik **30**, 408–424 (1931).

[10] C. Jensen, Meteorolog. Zeits. **49**, 419–430 (1932).

and in the plant, whenever visual determinations should be replaced by permanent records, for instance in connection with *sugar analysis, biological, and medical work*, etc. Other applications are pyrometric, whiteness, and opacity determinations, and general photometric work. This instrument may also help in the difficult work with the magneto-optic method of Allison.[11]

V. Summary

A modified Martens polarization photometer is connected with a motion-picture camera. The Nicol prism rotates, and the photometric field is photographed every ten degrees. The position of the Nicol prism is indicated on each record. The intersection of the density curves for the two halves of the photometric field is determined. The instrument may be used for measuring the polarization of light sources, and for density and transmission determinations in photometric and polarimetric work. An application is presented for the rapid determination of the polarization of the entire vault of the sky.

[11] F. Allison, Phys. Rev. **30**, 66–70 (1927).

Acknowledgments

Part of the costs of building the first instrument were defrayed by a grant from the American Association for the Advancement of Science, and by a loan from the Sidney Hellman Ehrman Fund of the University of California. Mr. Edward J. Hoff of Berkeley, California, helped in building the first instrument and Mr. George F. Tauchmann of Berkeley, California, devoted much care and skill completing the final arrangement. I am further obliged to Mr. Gilbert G. Hudson of Oakland, California, for help in building a photoelectric densitometer.

The Dielectric Bolometer

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(Received March 7, 1938)

1. INTRODUCTION

IN the bolometer as developed by Langley[1] in 1881, radiant power was measured by the unbalance of a direct-current Wheatstone bridge. The device was capable of the same order of sensitivity as that obtainable with the radiation thermocouple. But since the bolometer seemed to have no advantages over the thermocouple and required, moreover, additional auxiliary equipment, it never came into common use.

There are, however, two obvious extensions to the Langley bolometer:

(a) Operation on alternating current instead of direct current.

(b) Utilization of materials having a greater change in electrical properties with temperature.

The former development was investigated several years ago[2] and resulted in a dependable, rugged, and easily operated device. But in common with all nonselective radiometers, the a.c. metallic bolometer had a sensitivity* of a much lower order of magnitude in the visible region than the photoelectric cell. It was with the idea of raising the sensitivity that we undertook an investigation of (b), the preliminary results of which are described in this paper.

The sensitivity of the a.c. metallic bolometer[2, 3] is limited by two factors—the low temperature coefficient of resistance obtainable with metals, and the low resistance of the metallic elements, which are therefore not well matched to the input impedance of an a.c. amplifier. Bolometer elements of higher resistance have been made by Dewhurst[4] who used sputtered metal films. The resistance, however, was still of the wrong order of magnitude and the sputtered films had a temperature coefficient even lower than the 0.004 obtainable with pure metals in bulk. The idea developed in the present paper is to utilize the high impedance and the large temperature change in impedance known to exist in insulating materials, by employing these materials instead of metals in a *dielectric bolometer*.

Two identical thin plates of dielectric with thin electrodes are used in two arms (1 and 2, Fig. 1) of an a.c. bridge. The ratio arms consist of two equal impedances, 3 and 4. An alternating e.m.f. is applied to the bridge, and the amplifier input is connected across the high resistance Z_g. The amplifier output operates a telephone head-set or other null detector. When radiant energy is incident on 2, the temperature of the dielectric rises, its impedance changes, and the bridge is thrown out of balance. The irradiation of 2 is measured by noting the change which must be made in the parameters of one of the bridge arms to re-establish balance.

2. THEORY OF THE BOLOMETER BRIDGE

The first step in the present investigation is the development of a theory of the dielectric bolometer. The principal assumption made in the analysis is that the heating effect of the applied voltage is negligible, an assumption that appears to be justified for all ordinary dielectrics at room temperature. The following notation is used:

A = area of one electrode (cm²)
a_t = total absorption factor of front electrode (the receiving surface) for the kind of radiation being measured (numeric)
$B = Z_3/(Z_1+Z_3)$ (numeric)
$\mathfrak{C}$ = capacitance of one arm of the bridge (farad)
e_{t1} = total radiation factor* of front electrode (the receiving surface) for temperature T_a (numeric)
e_{t2} = total radiation factor of rear electrode for temperature T_a (numeric)
f = frequency (hertz)
G = irradiation* of front electrode (watt cm⁻²)
$G_{\min}$ = minimum detectable irradiation (watt cm⁻²)
$\mathfrak{G}$ = equivalent parallel conductance of one arm of the bridge (mho)
i = current (amp.)
$K = \dfrac{V_1 B}{4sT_a^3}\left(\dfrac{a_t}{e_{t1}+e_{t2}}\right)$
l = thickness of the dielectric plate (cm)

* The term "sensitivity" is here used in the same sense as in two previous papers (2 and 3 of the Bibliography); i.e., as the volts produced across the amplifier input per unit irradiation. Other interpretations of "sensitivity" are discussed in Section 9 of the present paper.

* The terminology is in accordance with Moon, *Scientific Basis of Illuminating Engineering* (New York, 1936).

R_d = equivalent series resistance of the dielectric bolometer element (ohm)
s = Stefan-Boltzmann constant 5.709×10^{-12} (watt cm^{-2} deg.$^{-4}$)
T = temperature of dielectric (°K)
T_a = ambient temperature (°K)
$\Delta T = T - T_a$
V = voltage applied to bridge (volt)
V_1 = voltage across arm 1 of bridge (volt)
V_g = voltage across amplifier input (volt)
$V_{\min}$ = minimum detectable voltage across amplifier input (volt)
$\mathbf{Z}$ = impedance (ohm $\angle$)
Z = magnitude of impedance (ohm)
z = bolometer sensitivity (volt watt^{-1} cm^2)
z_c = bolometer sensitivity when $\gamma_r = 0$ (volt watt^{-1} cm^2)
z_r = bolometer sensitivity when $\gamma_c = 0$ (volt watt^{-1} cm^2)
α_c = temperature coefficient of capacitance of the dielectric bolometer element at temperature T_a (° K^{-1})
α_r = temperature coefficient of conductance of the dielectric bolometer element at temperature T_a (° K^{-1})
$\boldsymbol{\gamma}_z$ = relative change in impedance of the dielectric bolometer element, caused by irradiation (complex number)
γ_c = relative change in capacitance of the dielectric bolometer element, caused by irradiation (numeric)
γ_r = relative change in conductance of the dielectric bolometer element, caused by irradiation (numeric)
δ = imperfection of angle of the dielectric (radians)
$\varkappa$ = relative permittivity of the dielectric (numeric)

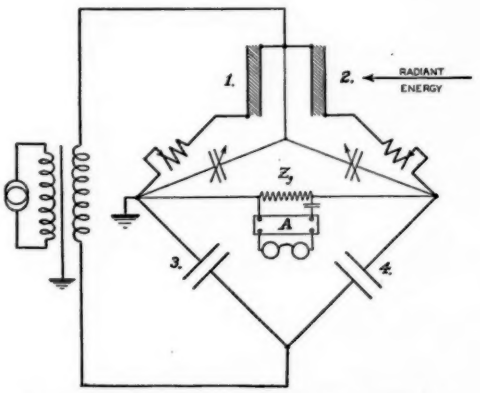

FIG. 1. Schematic diagram of a bolometer bridge.*

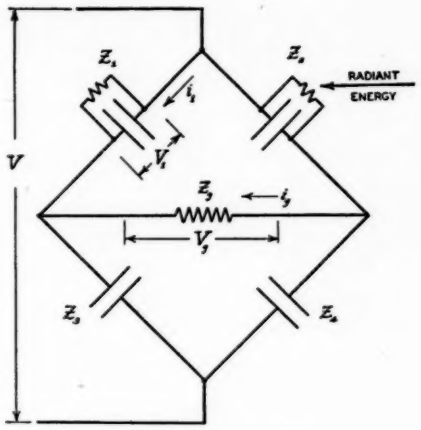

FIG. 2. Equivalent circuit.

σ = conductivity of dielectric (ohm^{-1} cm^{-1}) at temperature T
σ_a = conductivity of dielectric (ohm^{-1} cm^{-1}) at temperature T_a
$\Phi_{\min}$ = minimum detectable radiant power (watt)
$\omega = 2\pi f$.

Figure 2 gives the equivalent circuit for the complete bolometer bridge. $\mathbf{Z}_1$ and $\mathbf{Z}_2$ represent the total impedances of arms 1 and 2, including the small variable capacitors and resistors used for balancing (Fig. 1). In practice, the latter are made very small, so $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are essentially the impedances of the two bolometer elements. Arm 2 is irradiated. With the bridge balanced for no irradiation, both elements are assumed to be at the ambient temperature T_a, and $\mathbf{Z}_1 = \mathbf{Z}_2$, $\mathbf{Z}_3 = \mathbf{Z}_4$.

When arm 2 is irradiated, its temperature rises and its impedance changes from $\mathbf{Z}_2 = \mathbf{Z}_1$ to

$$\mathbf{Z}_2 = \mathbf{Z}_1(1-\boldsymbol{\gamma}_z), \tag{1}$$

where $\boldsymbol{\gamma}_z$ is a small complex quantity. The voltage across the amplifier is

$$\mathbf{V}_g = \mathbf{i}_g\mathbf{Z}_g. \tag{2}$$

A consideration of the circuital equations of the network gives

$$\mathbf{V}_g = \frac{\mathbf{V}\mathbf{Z}_1\mathbf{Z}_3\mathbf{Z}_g\boldsymbol{\gamma}_z}{(\mathbf{Z}_1+\mathbf{Z}_3)[\mathbf{Z}_3\mathbf{Z}_g+\mathbf{Z}_1(2\mathbf{Z}_3+\mathbf{Z}_g)(1-\boldsymbol{\gamma}_z)]+\mathbf{Z}_1\mathbf{Z}_3^2\boldsymbol{\gamma}_z}. \tag{3}$$

* The actual bridge used in practice may differ considerably from the schematic diagram of Fig. 1. The best bridge for a given set of conditions will depend on the characteristics of the bolometer dielectric and on various other factors.

The γ_s can be neglected when it appears in the denominator, and $\mathbf{Z}_g$ can be made large in comparison with $\mathbf{Z}_3$. Thus Eq. (3) becomes

$$\mathbf{V}_g \cong \mathbf{V}\mathbf{Z}_1\mathbf{Z}_3\gamma_s/(\mathbf{Z}_1+\mathbf{Z}_3)^2. \tag{3a}$$

High sensitivity is obtained by using high bridge voltage $\mathbf{V}$. But the voltage that can be applied to the bridge is limited, not by anything associated with the supply but by the breakdown voltage of the dielectric in arms 1 and 2. Thus it is convenient to express Eq. (3a) in terms of the voltage $\mathbf{V}_1$ across arm 1:

$$\mathbf{V}_1=\mathbf{i}_1\mathbf{Z}_1=\frac{(\mathbf{V}-\mathbf{i}_g\mathbf{Z}_3)\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_3}\cong\mathbf{V}\frac{Z_1}{Z_1+Z_3}. \tag{4}$$

Substitution in (3a) gives

$$\mathbf{V}_g=B\mathbf{V}_1\gamma_s, \tag{3b}$$

where

$$B=Z_3/(Z_1+Z_3). \tag{5}$$

For the special case of an equal-arm bridge with $Z_1 \cong Z_3$ and $Z_g \gg Z_1$, Eq. (5) shows that $B=\frac{1}{2}$. For greatest response, Z_3 is made very large with respect to Z_1. If $Z_3 \gg Z_1$ and $Z_g \gg Z_1$, then $B=1$. For other combinations of bridge parameters, the values of B are obtained from Eq. (5).

The symbols in bold-face type represent complex quantities. The equations may be put in usable form by making the following substitutions:

$$\mathbf{Z}_1=\frac{1}{\mathfrak{G}_1+j\omega\mathfrak{C}_1},$$

$$\mathbf{Z}_2=\frac{1}{\mathfrak{G}_1(1+\gamma_r)+j\omega\mathfrak{C}_1(1+\gamma_c)}, \tag{6}$$

$$\mathbf{Z}_3=1/j\omega\mathfrak{C}_3,$$

where $\mathfrak{G}_1$ = equivalent parallel conductance of arm 1 at temperature T_a

$\mathfrak{C}_1$ = equivalent parallel capacitance of arm 1 (farads)

$\mathfrak{C}_3$ = capacitance of arm 3. (Arms 3 and 4 are assumed to have negligible loss.)

γ_r = relative change in conductance of dielectric, caused by irradiation

γ_c = relative change in capacitance of dielectric, caused by irradiation.

But $\mathbf{Z}_2$ is also represented by

$$\mathbf{Z}_2=\mathbf{Z}_1(1-\gamma_s)=\frac{1}{\mathfrak{G}_1+j\omega\mathfrak{C}_1}(1-\gamma_s),$$

so, for small γ's

$$\gamma_s=\frac{(\mathfrak{G}_1^2\gamma_r+\omega^2\mathfrak{C}_1^2\gamma_c)+j\omega\mathfrak{C}_1\mathfrak{G}_1(\gamma_c-\gamma_r)}{\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2}. \tag{7}$$

The substitution of Eq. (7) in Eq. (3b) gives

$$\mathbf{V}_g=\frac{\mathbf{V}_1B}{\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2} \times[(\mathfrak{G}_1^2\gamma_r+\omega^2\mathfrak{C}_1^2\gamma_c)+j\omega\mathfrak{C}_1\mathfrak{G}_1(\gamma_c-\gamma_r)]. \tag{8}$$

Now suppose that the capacitance of the dielectric specimen is independent of temperature. Then $\gamma_c=0$, and (8) reduces to

$$\mathbf{V}_{gr}=\frac{\mathbf{V}_1B\mathfrak{G}_1\gamma_r}{\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2}[\mathfrak{G}_1-j\omega\mathfrak{C}_1]. \tag{9}$$

The magnitude of the voltage across the amplifier input is therefore

$$V_{gr}=\frac{V_1B\mathfrak{G}_1}{(\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2)^{\frac{1}{2}}}\gamma_r=\frac{V_1B\gamma_r}{(1+1/\tan^2\delta)^{\frac{1}{2}}}, \tag{10}$$

where δ = imperfection angle of the dielectric, and

$$\tan\delta=\mathfrak{G}_1/\omega\mathfrak{C}_1.$$

Eq. (10) applies even if the capacitance is a function of temperature, provided the change is compensated by the adjustment of the small auxiliary variable air condenser in the No. 2 bolometer arm.

If, conversely, the change of capacitance is used as the measure of the irradiation, and the conductance change is compensated by the use of a variable resistor in the No. 2 arm, then $\gamma_r=0$ and

$$\mathbf{V}_{gc}=\frac{\mathbf{V}_1B\omega\mathfrak{C}_1\gamma_c}{\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2}[\omega\mathfrak{C}_1+j\mathfrak{G}_1]. \tag{11}$$

The magnitude of the voltage is

$$V_{gc}=\frac{V_1B\omega\mathfrak{C}_1\gamma_c}{(\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2)^{\frac{1}{2}}}=\frac{V_1B\gamma_c}{(1+\tan^2\delta)^{\frac{1}{2}}}. \tag{12}$$

3. Theory of the Complete Bolometer

The temperature rise of the irradiated arm must now be evaluated in terms of the irradiation. The radiant power absorbed by the re-

ceiving electrode of area A is

$$P_a = a_t G A.$$

If conduction and convection heat losses are reduced to negligible values, the total power lost by the dielectric is

$$P_r = e_{t1} s(T^4 - T_a^4)A + e_{t2} s(T^4 - T_a^4)A.$$

For equilibrium, $P_a = P_r$, and

$$a_t G = s(T^4 - T_a^4)(e_{t1} + e_{t2}). \tag{13}$$

But

$$T^4 = T_a^4 + 4T_a^3 \cdot \Delta T + \cdots.$$

Since ΔT is always very small, higher powers may be neglected. Eq. (13) then becomes

$$\Delta T = \frac{G}{4sT_a^3}\left(\frac{a_t}{e_{t1}+e_{t2}}\right). \tag{14}$$

Conductance variation ($\gamma_c = 0$)

At any temperature T near T_a, the conductivity of the dielectric in arm 2 is

$$\sigma = \sigma_a[1 + \alpha_r(T - T_a)] (\text{ohm}^{-1}\ \text{cm}^{-1}). \tag{15}$$

The corresponding conductance is

$$\mathfrak{G}_2 = \mathfrak{G}_1[1 + \alpha_r \cdot \Delta T].$$

But $\mathfrak{G}_2 = \mathfrak{G}_1(1 + \gamma_r)$, so

$$\gamma_r = \alpha_r \cdot \Delta T. \tag{16}$$

Substitution from Eq. (14) gives

$$\gamma_r = \frac{G}{4sT_a^3}\left(\frac{a_t}{e_{t1}+e_{t2}}\right)\alpha_r. \tag{17}$$

The magnitude of the voltage appearing across the amplifier input is obtained by substituting Eq. (17) in (10), or

$$V_{gr} = \frac{GK\alpha_r}{(1 + 1/\tan^2\delta)^{\frac{1}{2}}} \text{ (volts)}, \tag{18}$$

where

$$K = \frac{V_1 B}{4sT_a^3}\left(\frac{a_t}{e_{t1}+e_{t2}}\right). \tag{19}$$

If the bolometer sensitivity z_r is defined as the ratio of detector voltage to irradiation, then

$$z_r = \frac{V_{gr}}{G} = \frac{K\alpha_r}{(1 + 1/\tan^2\delta)^{\frac{1}{2}}} \text{ (volt watt}^{-1}\ \text{cm}^2). \tag{20}$$

With ordinary commercial dielectrics at room temperature, $\tan\delta$ is always small compared with unity, so for such cases Eq. (20) assumes the simple form:

$$z_r \cong K\alpha_r \tan\delta. \tag{20a}$$

Equations (20) and (19) show that the bolometer sensitivity varies directly with the voltage applied to the dielectric specimen, is independent of the area of the electrodes, and varies directly with the temperature coefficient α_r. For a nonselective electrode material and with the same material for both electrodes,

$$a_t = e_{t1} = e_{t2}$$

and

$$K = V_1 B / 8sT_a^3. \tag{19a}$$

Equation (19a) indicates that on the assumption of no loss of heat by convection and conduction and with a nonselective electrode material, the sensitivity is *independent of the electrode material.*

The sensitivity can be almost doubled by using a black receiving face and a bright rear electrode. Then e_{t2} can be made practically negligible in comparison with e_{t1}, and Eq. (19) becomes

$$K = V_1 B / 4sT_a^3. \tag{19b}$$

Another way of increasing z is to use unblackened metallic surfaces or other selectively radiating surfaces. Then a_t, which refers to the received wave-lengths in the visible region, may be as high as 0.50 while e_{t1} and e_{t2}, which refer to the radiated wave-lengths in the neighborhood of $\lambda = 10\mu$, may be of the order of 0.01.

Capacitance variation ($\gamma_r = 0$)

Let the relative permittivity (dielectric constant) at temperature T be

$$\varkappa = \varkappa_a[1 + \alpha_c(T - T_a)] \text{ (numeric)}. \tag{21}$$

The capacitance of the irradiated arm is

$$\mathfrak{C}_2 = 8.842 \times 10^{-14} \varkappa (A/l) = \mathfrak{C}_1[1 + \alpha_c \cdot \Delta T] \text{ (farads)}.$$

But $\mathfrak{C}_2 = \mathfrak{C}_1(1 + \gamma_c)$,

so $\gamma_c = \alpha_c \cdot \Delta T$

or

$$\gamma_c = \frac{G}{4sT_a^3}\left(\frac{a_t}{e_{t1}+e_{t2}}\right)\alpha_c. \tag{22}$$

Substitution in Eq. (12) gives

$$V_{gc}=KG\alpha_c/(1+\tan^2\delta)^{\frac{1}{2}} \tag{23}$$

or the bolometer sensitivity is

$$z_c=K\alpha_c/(1+\tan^2\delta)^{\frac{1}{2}}. \tag{24}$$

For the special case of $\tan^2\delta \ll 1$,

$$z_c \cong K\alpha_c. \tag{24a}$$

4. Dielectrics

An ideal dielectric for use in the bolometer should have the following properties:

1. Large change in either dielectric constant or equivalent conductivity with change in temperature.
2. Small temperature change in the parameter which is not being used in the measurements.
3. Permanence, ability to give reproducible results.
4. Availability in thin sheets.
5. Ability to stand high temperatures.

Item (2) is by no means a necessity, but the ease of operation of the bridge is enhanced if the quantity not being read does not change to a marked extent. Item (4) is necessary for two reasons. According to theory, the sensitivity of the bolometer is unaffected by change in thickness; but obviously the speed of response is greatly affected. Also, thin specimens are necessary in order to reduce the impedance to an easily measurable value. Item (5) is helpful because of the sealing-in and evacuation of the bolometer cell. It is somewhat questionable if a sufficiently good vacuum can be maintained if the bolometer elements and enclosing glass globe cannot be heated at the time of evacuation.

Table I. *Unpolarized conductivity*

Material	Reference	σ_1	β_1
Soda-lime glass (Kimble Glass Co.)	6 5	53.1	9,950 9,220
Pyrex chemical-resistant glass (Corning Glass Works)	6 5	24.2	10,900 10,100
G-1 lead glass (Corning Glass Works)	6 5	1,810	14,100 12,400
Fused quartz (The Thermal Syndicate)	6 5	71.4	16,300 13,800
NaCl	17		9,550
Muscovite mica	17		8,820

A number of insulating materials is available having a wide range of properties. Of inorganic materials, one naturally thinks of crystals such as rocksalt, quartz, mica, several metallic oxides, fused quartz, and various glasses. In the organic field, there are such materials as paper, cloth, cellulose acetate, Cellophane, celluloid, rubber, collodion, gelatine, varnish films, etc. A brief summary of data on some of these dielectrics will now be given.

The d.c. unpolarized* conductivity of most dielectrics satisfies the empirical equation

$$\sigma=\sigma_1 e^{-\beta_1/T}\ (\text{ohm}^{-1}\ \text{cm}^{-1}), \tag{25}$$

where σ_1 and β_1 are constants. Eq. (25) has been found to apply for a large number of dielectrics and over a surprisingly wide temperature range—at least from 0°C to a temperature at which the material softens, chars, or otherwise changes its properties in a marked manner. For all these materials, β_1 is found to be in the neighborhood of 10,000°K as shown in Table I.

For a given sample of dielectric operated at constant voltage, the unpolarized, direct-current dielectric loss varies directly with σ, Eq. (25), and increases rapidly as the temperature is raised. This conduction loss is present in unchanged magnitude[5] if the sample of dielectric is operated on alternating current instead of direct current, the r.m.s. voltage being the same in both cases. At high temperatures, the conduction loss is so large compared with any other a.c. loss which may be present that the apparent a.c. conductivity is essentially the same as the d.c. conductivity. This condition is shown in Fig. 3 for a soda-lime glass tested at Massachusetts Institute of Technology.

At temperatures above approximately 180°C, the a.c. conductivity is seen to coincide with the d.c. conductivity. At lower temperatures, however, the d.c. conductivity continues along the straight line representing Eq. (25), while the a.c. curve gradually departs from the straight line. This fact indicates that on alternating current there exists a component of loss not caused by electric conduction. At high temperatures this component is masked by the large conduction loss, but at lower temperatures it assumes the

* Electrolytic polarization or space-charge effect eliminated by any one of several methods (references 5 and 6).

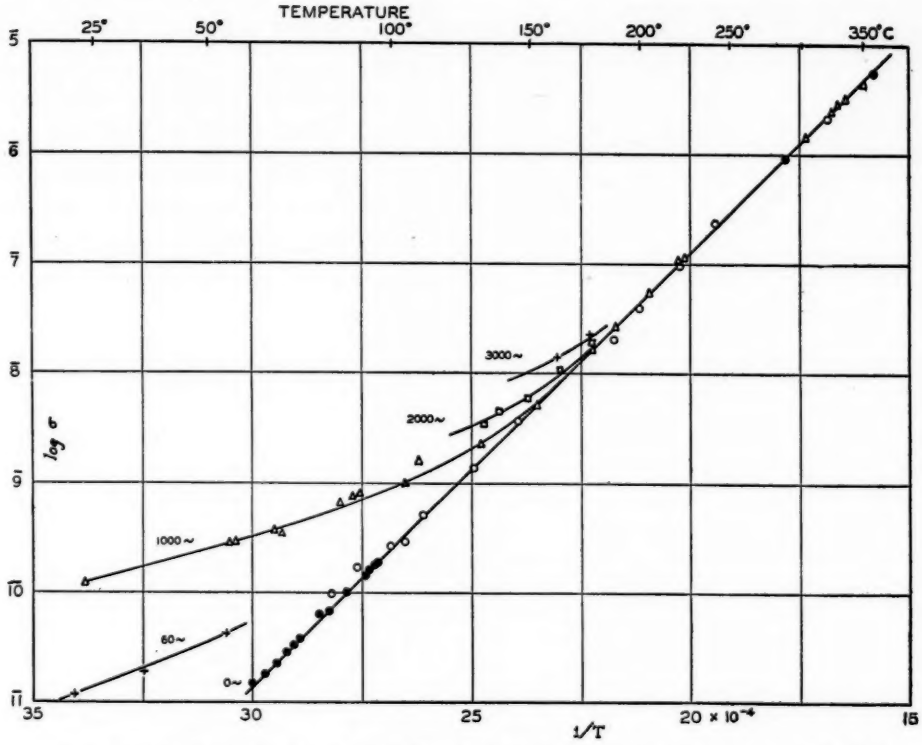

FIG. 3. Unpolarized conductivity of a soda-lime glass. Cross, alternating current, $f=60$, D. M. Robinson. Triangle, alternating current, $f=1000$, D. M. Robinson. Open circle, direct current, initial-instant method, D. M. Robinson. Solid circle, direct current, nonpolarizing electrodes. High temps., V. G. Mukhdjian. Low temps., Parry Moon.

major role. The additional component has been recognized since the time of Hopkinson and has been studied by Wagner,[9] Tank,[10] and others but without the invention of any satisfactory theory. Fig. 3 shows also that as the frequency is raised the a.c. loss increases and the departure of the curve from the d.c. line occurs at higher temperatures. Other glasses have similar curves.[5]

The permittivity of glasses changes very little with temperature, the curves being almost straight with a slight positive slope from −180° to +100 or 200°C, above which the slope increases.[11] Evidently if glass is to be used in a dielectric bolometer, capacitance variation will give a rather low sensitivity while resistance variation will give highest sensitivity only at a very low frequency or at a high temperature where the large slope of the d.c. characteristic can be approached.

Many other inorganic dielectrics behave in a similar manner, but a distinctly different characteristic is found for materials having polar molecules. Fig. 4 shows typical curves calculated by the Debye theory. Tests have shown the theory to be in remarkably good agreement with experiments for a number of materials such as rosin and rubber-sulphur compounds. The principal features of the curves are a sharp rise in permittivity, coupled with a peak in the equivalent conductivity curve. The curves can be moved along the temperature axis by changing the frequency. At high temperatures, the conduction in the dielectric (which is neglected in the Debye theory) becomes important, and the experimental σ-curves exhibit another rise not shown in Fig. 4.

Figures 5 and 6 give experimental results on celluloid and cellulose acetate. It is seen that the

curves are in qualitative agreement with the theoretical curves of Fig. 4. An increase in frequency will move the curves to the right and may even allow placement of the peak of the tan δ curves at room temperature.

5. Calculated Sensitivity

Values of the temperature coefficients α_r and α_c have been calculated from various data and are assembled in Table II. It should be realized, of course, that slight differences in the chemical composition of the materials or in test procedure may cause decided changes in the results. On the basis of Table II, however, values of bolometer sensitivity have been calculated and are given in Table III. These values are for $V_1=100$ volts, $T_a=293°$K, $f=1000$, $a=e_{11}=e_{12}$, $Z_g \gg Z_1$, $Z_3=Z_1$. By Eq. (19a), $K=4.36\times10^4$. Calculations are then made by use of Eqs. (20a) and (24a) but with an additional 10 percent reduction in z to compensate for reflection losses at the glass globe. Evidently the values of Table III should be easily attainable in practice, and values of z several times as high should be possible by the use of high voltage, by increasing Z_3 so $Z_3 \gg Z_1$, by the use of bright rear electrodes instead of black ones, or by the operation on more favorable parts of the characteristics resulting from a change of temperature or frequency.

Table III indicates that with these dielectrics operated at a frequency of 1000 and at room temperature, the sensitivity z_c obtained by capacitance variation is greater in nearly every case than the sensitivity z_r obtained by conductance variation. It will be noted also that in even the worst cases the values are high compared with the best sensitivities obtained with thermocouples or other nonselective radiometers (Table V). They are all low, however, in comparison with the theoretically possible sensitivity obtainable by use of the d.c. conductivity curve. For a dielectric obeying Eq. (25),

$$\alpha_r=\beta_1/T_a^2 \tag{26}$$

and thus,

$$z_r=\frac{K\beta_1}{T_a^2(1+1/\tan^2\delta)^{\frac{1}{2}}}=\frac{K\beta_1\mathfrak{G}_1}{T_a^2(\mathfrak{G}_1^2+\omega^2\mathfrak{C}_1^2)^{\frac{1}{2}}}. \tag{20b}$$

If, because of low frequency or high conductivity, $\mathfrak{G}_1^2 \gg \omega^2\mathfrak{C}_1^2$,

$$z_r=K\beta_1/T_a^2 \tag{20c}$$

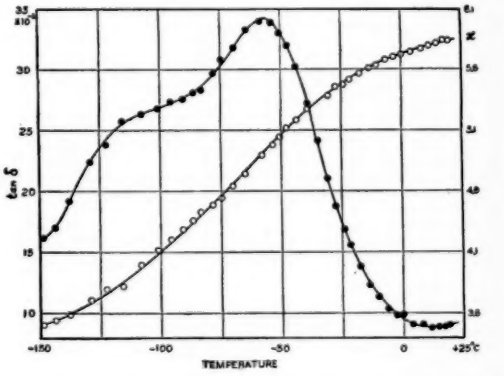

Fig. 5. Dielectric properties of cellulose nitrate. $f=1000$, thickness $=250\mu$. Solid circle = tan δ. Open circle = relative permittivity.[8]

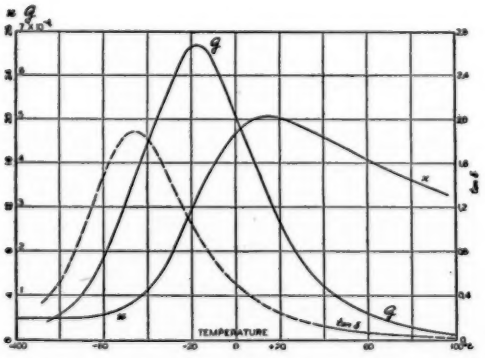

Fig. 4. Typical calculated curve for a dielectric composed of polar molecules.

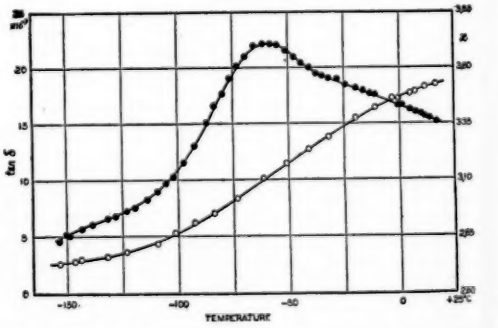

Fig. 6. Dielectric properties of cellulose acetate. $f=1000$, thickness $=120\mu$. Solid circle = tan δ. Open circle = relative permittivity.[8]

and for $V_1=100$, $\beta_1=10{,}000$, $B=1$, $T_a=293$, $a=e_{t1}=0.98$, $e_{t2}=0.02$,

$$K=1.71\times10^5, \quad z_r=20{,}000.$$

6. Construction

The foregoing calculations show that surprisingly high values of sensitivity are obtainable with the dielectric bolometer. Theory, however, can tell very little about practical difficulties that may conceivably limit the actual attainment of such values of sensitivity. A number of bolometer cells were constructed, therefore, and tests were made. The results available at the present time by no means constitute a thorough investigation of bolometer possibilities but they do indicate that high values of z can be obtained and that measurements can be made with considerable accuracy and speed.

Two materials, commercially available in very thin sheets, were used: Cellophane of 22-micron (0.9×10^{-3} in.) thickness, and Pliofilm (a rubber hydrochloride) of 25-micron thickness.

Probably the best electrodes would be gold or platinum, applied either by sputtering or by evaporation. Blackening could be provided by the evaporation of bismuth black, as was done with the targets of the a.c. metallic bolometer.[3] In the present bolometers, however, it was found that Aquadag electrodes were easier to apply and gave satisfactory results. Thinned to the consistency of medium oil, the Aquadag was applied with ruling pen or brush and gave a fairly uniform layer of graphite. The layer was found to be of low resistance compared with the resistance of the dielectric and was reasonably dark (reflection factor 0.135 throughout the visible spectrum) so that no further blackening was used.

The actual construction is shown in Figs. 7 and 8. A single piece of dielectric was employed for the two bolometer elements, and electrodes were applied as indicated in Fig. 7. Each active element was 32 mm×12.5 mm, giving an area of 4 cm². Contact to the graphite electrode was made by thin tinfoil strips fastened in place with Aquadag. The framework (4, Fig. 8), of bright nickel, supported the dielectric and shielded one of the elements from radiation.

The bolometer elements and their support

Table II. *Properties of some insulating materials* ($f=1000\sim$).

Material	20°C						30°C					Authority
	κ	α_c	σ	α_r	tan δ	κ	α_c	σ	α_r	tan δ	
Glasses												
Soda-lime	6.3	—	1.2×10^{-10}	28×10^{-3}								Robinson [5]
Pyrex	4.9	—	1.2×10^{-11}	18	$8.2^*\times10^{-3}$							Robinson [5]
Lead (G-1)	—	—	2×10^{-11}	14	1.0†							Allia [6]
Micro. cover	7.4	1.0×10^{-3}	8.2×10^{-11}	18	20	—	—	9.7×10^{-11}	16×10^{-3}	—		Fleming & Dyke [16]
Micro. cover	6.5	0.92	6.2×10^{-11}	21	17	6.6	1.4×10^{-3}	7.6×10^{-11}	21	20×10^{-3}		
Muscovite Micas												B & M [17]
India	6.5	0.10	1.4×10^{-13}	37	<0.1	6.5	0.10	—	34	—		Fleming & Dyke
India	4.0	0.62	2.5×10^{-12}	10	1.2	4.0	0.62	2.8×10^{-12}	10	1.6		Allia
India Green	6.3	0.04	7.4×10^{-13}	12	0.2	—	—	8.3×10^{-13}	12	—		
Phlogopite Micas												Allia
Madagascar Green (*A*)	5.8	0.41	2.6×10^{-11}	33	8.0	—	—	3.5×10^{-11}	31	—		Allia
Madagascar Green (*E*)	5.8	0.18	8.7×10^{-12}	26	2.7	—	—	1.1×10^{-11}	24	—		Allia
Madagascar Green (*F*)	6.1	0.38	1.4×10^{-11}	34	4.2	—	—	2.0×10^{-11}	32	—		Allia
Madagascar Green (*H*)	6.2	0.30	1.9×10^{-11}	40	5.4	—	—	2.8×10^{-11}	37	—		Allia
Madagascar Green (*I*)	6.5	0.22	8.7×10^{-12}	39	2.4	—	—	1.3×10^{-11}	37	—		Allia
Madagascar Green (*J*)	5.8	0.17	5.8×10^{-12}	34	1.8	—	—	8.0×10^{-12}	31	—		Allia
Madagascar Brown	5.2	0.06	5.0×10^{-11}	38	17	—	—	7.2×10^{-11}	35	—		Allia
	4.3	0.30	$<10^{-14}$	111	<0.1	4.3	0.30	$<10^{-14}$	104	<0.1		B & M
Rocksalt (NaCl)	5.8	0.79	3.0×10^{-11}	39	9.1	—	—	—	—	—		Allia
Celluloid	4.4	3.7	7.1×10^{-11}	63	28	4.5	3.7	13.5×10^{-11}	63	30		Fleming & Dyke
	4.4	3.7	3.1×10^{-11}	44	13	4.5	3.7	4.9×10^{-11}	44	20		Fleming & Dyke
Cellulose Acetate	3.5	0.90	2.9×10^{-11}	38	15	—	—	—	—	—		Allia
Ebonite	3.2	+1.1	7.2×10^{-12}	21	4.1	—	—	9.6×10^{-12}	39	7.1		Fleming & Dyke
Vulcanized Rubber	2.7	−2.9	2.9×10^{-12}	0	2.0	—	−3.0	4.1×10^{-12}	+63	3.0		Fleming & Dyke
Gutta-percha	2.9	+1.1	1.9×10^{-11}	−100	12	—	+1.1	8.3×10^{-12}	−61	6.0		Fleming & Dyke

* McDowell and Begeman, Phys. Rev. **31**, 476 (1928). † E. Schott.

were sealed into a glass bulb, as in the construction of radio tubes. With these inflammable dielectrics, however, it was found necessary to invert the tube in the sealing-in process. The bolometer elements were then below the flames and heating of the dielectric by convection was eliminated. Evacuation was accomplished with an oil pump. A getter was used but no serious attempt was made to outgas the metal structure or the walls of the envelope, since such procedure would certainly injure the dielectric. Twenty-one satisfactory cells were constructed in this way.

The remainder of the bridge was easily constructed. Arms 3 and 4 (Fig. 1) had capacitances of approximately 500$\mu\mu$f each. Small variable air capacitors (15$\mu\mu$f) and resistors (1500 ohms) were used in balancing the bridge. Each arm of the bridge was in a separate metal shield and the entire assembly was housed in a grounded metal case. Two stages of amplification were arranged in the Johnson and Neitzert manner.[20] The complete bolometer assembly measured 18×20×38 cm and contained a bolometer cell, bolometer bridge, amplifier, and necessary B and C batteries.

TABLE III. *Calculated bolometer sensitivity. 20°C, 1000∼, $K = 1.75 \times 10^5$.*

Material	Authority	Sensitivity (volt watt^{-1} cm^2) z_e	z_r
Glass			
Micro. cover	Allia	39	14
Micro. cover	Fleming & Dyke	36	14
Muscovite Micas			
India	B. & M.	4	0.07
India	Fleming & Dyke	25	0.5
India Green	Allia	2	0.09
Phlogopite Micas			
Madagascar Green (*A*)	Allia	16	10
Madagascar Green (*E*)	Allia	11	3
Madagascar Green (*F*)	Allia	7	6
Madagascar Green (*H*)	Allia	11	9
Madagascar Green (*I*)	Allia	8	4
Madagascar Green (*J*)	Allia	7	2
Madagascar Brown	Allia	3	2
Rocksalt		11	<0.4
Celluloid	Allia	31	14
	Fleming & Dyke	135	70
	Fleming & Dyke	135	23
Cellulose Acetate	Allia	35	23
Ebonite	Fleming & Dyke	41	3
Vulcanized Rubber	Fleming & Dyke	106	0
Gutta-percha	Fleming & Dyke	41	47

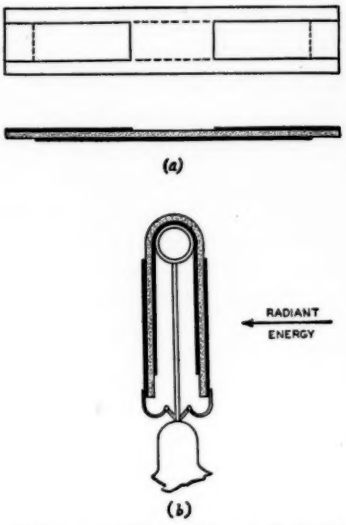

FIG. 7. Bolometer construction. (a) Strip of dielectric with graphite electrodes applied to give two identical bolometer elements. (b) Cross section through assembly, showing dielectric strip bent over nickel support.

7. EXPERIMENTAL RESULTS

Before any cells were constructed, a few tests were made on the materials. A film of graphite, obtained by painting Aquadag on Cellophane, was tested for resistance to make sure that the voltage drop in the electrodes would not be excessive. The film was 12.5 mm wide and 4 cm long. The resistance from end to end was 550 ohms. The construction of the bolometer (Fig. 7) gave the same electrode resistance in series with each point of the dielectric, so the electrodes introduced a series resistance of approximately 550 ohms. Since the equivalent series resistance of the dielectric at room temperature was approximately 2000 ohms for Cellophane and 14,000 ohms for Pliofilm, the effect of the electrode resistance should not be troublesome.

The breakdown voltage of bolometer elements in air was measured on alternating current (60 hertz). The results were as follows:

Cellophane Elements	Pliofilm Elements
200 volts	300 volts
240	300
190	75
230	260
260	140
60	180

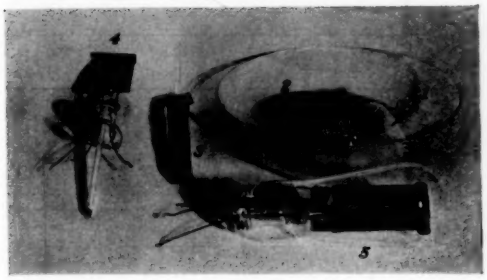

FIG. 8. Stages in the construction of a dielectric bolometer. (1) Strip of dielectric. (2) Aquadag. (3) Dielectric strip with electrodes, forming two bolometer elements. (4) Vacuum tube stem with nickel support. (5) Complete bolometer cell.

Evidently a value of $V_1=100$ volts could be used in most cases. It is interesting to note that an increase in the thickness of the dielectric would allow still greater voltages, with corresponding increase in sensitivity.

The completed and evacuated cells were tested on an a.c. bridge operated at 1000 hertz. The characteristics of a number of the cells are assembled in Table IV. These values give an idea of the variation obtained in these experimental cells. It will be noted that differences between the two elements in a given cell are usually less than 10 percent. The capacitance of four of these tubes were measured again after 90 days. The values were as follows:

No.	Element	Capacitance ($\mu\mu$f) Original	After 90 days
X2	A	445	418
X2	B	465	440
X3	A	450	438
X3	B	450	423
X4	A	485	450
X4	B	475	455
X5	A	475	435
X5	B	475	440

The capacitance decreased in every case. However, no appreciable change could be found in the sensitivity before and after aging.

Early tests indicated that the sensitivity of the Pliofilm bolometers was roughly ten times that of the Cellophane bolometers, and later work was confined to those made with Pliofilm. The following results were obtained on one of the Pliofilm bolometers made at a somewhat later date than the foregoing and having the following characteristics: $\mathfrak{C}=490$ and $485\mu\mu$f, $R_s=32{,}400$ and 32,800 ohms, tan $\delta=0.100$. The radiation source was a carbon incandescent lamp calibrated by the National Bureau of Standards. The bolometer target was 1.00 m from the lamp, at which distance the irradiation was 350×10^{-6} watt cm^{-2}. With $V_1=8.5$ volts, 1000 hertz, supplied by an oscillator, the voltage produced across the amplifier input by the irradiation of the target was found to be 4.4×10^{-3} volt. The sensitivity was therefore

$$z_c=12.6 \text{ volt watt}^{-1} \text{ cm}^2.$$

To make this value comparable with the theoretical values of Table III, we must reduce it to the common basis of $V_1=100$ volts. At this voltage, the sensitivity would be

$$z_c=12.6\times100/8.5=148 \text{ volt watt}^{-1} \text{ cm}^2.$$

These values are seen to be within the range calculated in the table.

Measurements on the speed of response gave a time-constant of $3\frac{1}{2}$ sec., which is low enough for most uses. In fact, a somewhat greater thickness of dielectric might still allow sufficiently rapid response and would also give in-

TABLE IV. *Properties of bolometer cells, 20°C, $\omega=6280$.*

Cell No.	Element	$\mathfrak{C}_s$	R_s	$\mathfrak{G}_p$	tan δ
Cellophane					
X7	a	1750×10^{-12}	1,900	2.30×10^{-7}	0.0210
X7	b	1800	1,900	2.43	0.0215
X8	a	1470	2,200	1.87	0.0203
X8	b	1440	2,400	1.96	0.0217
X12	a	1770	1,800	2.22	0.0200
X12	b	1700	1,900	2.16	0.0202
Pliofilm					
X1	a	420×10^{-12}	12,100	8.43×10^{-8}	0.0320
X1	b	410	12,800	8.49	0.0329
X2	a	445	13,200	10.3	0.0368
X2	b	465	13,700	11.7	0.0400
X3	a	450	12,400	9.90	0.0350
X3	b	450	14,100	11.2	0.0397
X4	a	485	13,100	12.1	0.0398
X4	b	475	14,400	12.8	0.0430
X5	a	475	15,900	14.1	0.0473
X5	b	475	15,900	14.1	0.0473

TABLE V. *Limiting sensitivities of various radiometers. Theoretical z.*

Device	z (volt watt^{-1} cm^2)	Detector	V_{min} (volt)	G_{min} (watt cm^{-2})	A (cm^2)	Φ_{min} (watt)
Constantan-copper vacuum-thermocouple	0.070	Galvanometer	10^{-10}	$3{,}000\times10^{-12}$	0.01	30×10^{-12}
Bismuth alloy vacuum-thermocouple	0.21	Galvanometer	10^{-10}	900	0.01	9
Tellurium-bismuth vacuum-thermocouple	0.70	a.c. amplifier	10^{-9}	70,000	0.01	700
a.c. metallic bolometer	>0.20	a.c. amplifier	10^{-8}	<50,000	0.01	<500
Dielectric bolometer	20,000	a.c. amplifier	10^{-7}	5	1	5
Experimental z						
Burt vacuum thermocouple	0.14	Galvanometer	10^{-10}	$1{,}000\times10^{-12}$	0.01	10×10^{-12}
a.c. metallic bolometer	0.068	a.c. amplifier	10^{-8}	200,000	0.01	2,000
Hayes receiver	126	a.c. amplifier	3×10^{-7}	2,000	10	20,000
Dielectric bolometer	148	a.c. amplifier	10^{-7}	700	1	3,000
Selenium barrier-layer cell ($\lambda=0.59\mu$)	2,000	Galvanometer	3×10^{-9}	1	10	10
Selenium barrier-layer cell ($\lambda=0.59\mu$)	2,000	a.c. amplifier	1.2×10^{-8}	1	10	10
Cs—O—Ag photo-cell ($\lambda=0.8\mu$, $R=10^{11}$ ohms)	4×10^{9}	d.c. amplifier	3×10^{-5}	0.01	10	0.1
Cs—O—Ag photo-cell ($\lambda=0.8\mu$, $R=10^{7}$ ohms)	4×10^{8}	a.c. amplifier	3×10^{-7}	0.1	10	1

creased sensitivity by allowing a correspondingly higher V_1.

8. Possibilities of Further Development

The foregoing treatment constitutes only a first step in the study of the dielectric bolometer. There is a wide variety of insulating materials which offer interesting possibilities, and each of these materials may be operated at various temperatures and frequencies and with various thicknesses of dielectric and various electrode materials to find optimum conditions. In the attempt to approach the theoretical maximum z more closely, one should not overlook the following possibilities:

1. Use polar dielectrics and adjust the frequency of the a.c. supply to give operation near the hump in the tan δ-curve. Examination of various curves indicates that temperature coefficients as high as $\alpha=0.05$ (ten times that obtainable with metals) may be possible. Whether the resulting instrument will be sufficiently stable for precision work can be determined only by experiment.

2. Glass and mica have such attractive mechanical and thermal properties that a serious attempt to investigate their bolometric potentialities is advisable. It is possible that conductance-variation with high α_r can be obtained at room temperature by the use of very low frequency or with special glasses having unusually high conductivity. Another possibility is to raise the ambient temperature. Such a step is impracticable in portable instruments but would not be particularly objectionable in laboratory instruments of the highest precision where some kind of temperature control is desirable in any case. With soda-lime glass (Fig. 3), for example, a frequency of 60, and a temperature of 100°C, $\sigma=3\times10^{-10}$ and $\beta_1=9220$. Then

$$\alpha_r=\beta_1/T_a^2=66.2\times10^{-3}.$$

For a relative permittivity of 6.3, the capacitance of a cm cube is

$$\mathfrak{C}=8.84\times10^{-14}\varkappa=5.57\times10^{-13}\text{ farad}$$

and the tangent of the imperfection angle is

$$\tan\delta=\mathfrak{G}/\omega\mathfrak{C}=1.43.$$

By Eq. (20), for $K=1.71\times10^5$,

$$z_r=K\alpha_r/(1+1/\tan^2\delta)^{\frac{1}{2}}=9270\text{ volt watt}^{-1}\text{ cm}^2.$$

3. Another possibility of increasing the sensitivity in a given band of wave-lengths (such as the visible region) is to use selective absorbing surfaces.[2]

9. Comparison of Radiometric Devices

A comparison of sensitivity values, experimental and calculated, for various radiometric devices is given in Table V. Some of the values are taken from a previous paper;[2] others are given here for the first time. It should be realized that such tabulated values are necessarily very rough approximations. Not only do the characteristics depend upon constructional

details and upon the skill and experience of the operator; but also in comparing devices of such different types, numerous assumptions of a rather arbitrary nature must be made. In Table V it is assumed that when a low resistance galvanometer is used with a low resistance source, one-half of the generated e.m.f. appears across the galvanometer; that if bridge circuits are used they are proportioned for maximum sensitivity; that when a flicker disk is employed it reduces the r.m.s. voltage to 20 percent the value without the disk.

A comparison of the calculated values of sensitivity z shows that the theoretical maximum for the dielectric bolometer is greatly superior to the theoretical maxima for any of the other nonselective radiometers. The superiority of the newer devices—the dielectric bolometer and the Hayes receiver—is shown also by the experimental values of z.

In the foregoing treatment the quantity z has been called the "sensitivity" of the device and has been used throughout as a criterion of radiometric performance. It gives the response of the radiometer, in volts, per unit irradiation. A number of other criteria may be used, however, depending upon conditions.

Radiant intensity or luminous intensity of lamps, for instance, may be measured by a radiometer set up at sufficient distance from the source so that the inverse-square law holds to within the limits of experimental error. Candlepower distribution curves are obtainable by using a radiometer fitted with a filter transmitting according to the standard visibility curve. The radiation field at the radiometer is sensibly constant over a distance of several cm perpendicular to the rays from the lamp, so a large receiving surface is allowable and desirable. It is in such applications that the dielectric bolometer with its inherently large target shows up to best advantage. The radiation thermocouple, on the other hand, is handicapped by the small amount of energy which its minute target can absorb. More energy can be abstracted from the field by employing a thermopile with a large number of junctions, but constructional difficulties become a limiting factor. Increasing the response by imaging the source on the thermocouple target is inadvisable in these applications, since differences in the physical dimensions of the sources under test and in their distances from the target are usually necessary, and any form of optical system tends to introduce uncertainties in the calibration.

Moreover, the radiant energy available in many engineering tests is sufficient so that the ultimate limitation imposed by Brownian motion need not be considered. In the radiometry of lamps, a 10-watt tungsten lamp at a distance of 1.5 m produces an irradiation of approximately 30×10^{-6} watt cm^{-2}. If a dielectric bolometer is used with $z=100$, the voltage across the amplifier input caused by a one-percent change in this irradiation is 30×10^{-6} volt. Even if an optical filter having a transmission of 0.01 is used between lamp and target, the voltage caused by a one-percent change in the resulting $G=0.30\times10^{-6}$ is still 30 times as large as the Brownian voltage in a one-megohm resistor and is several hundred times as great as the effect of tube noises in a well-designed amplifier. It can be concluded, therefore, that in many technical applications the values of z given in Table V are a true criterion of sensitivity.

In many other applications, such as spectroradiometry, the measurement of radiant energy from stars, etc., the quantity z becomes essentially meaningless. If the radiometer is immersed in a uniform radiation field which is so weak that the required precision can be obtained only by pushing the device to the limit, the minimum detectable irradiation $G_{\min}$ becomes a useful criterion. The ultimate limit is then imposed by Brownian motion. For a low resistance galvanometer this limit is of the order of 10^{-10} volt, while for a resistor in the grid circuit of a thermionic tube, it is[29]

$$V_{\min}=(\bar{e}^2)^{\frac{1}{2}}=(4kT)^{\frac{1}{2}}(R+R_0)^{\frac{1}{2}}(f_2-f_1)^{\frac{1}{2}} \quad (27)$$

where V = minimum detectable r.m.s. voltage, which is here assumed for simplicity to be equal to

$(\bar{e}^2)^{\frac{1}{2}}$ = r.m.s. voltage across resistor, caused by thermal agitation. Also
k = Boltzmann's constant $=1.37\times10^{-23}$,
R = resistance of resistor (ohms),
R_0 = noise caused by first tube, expressed as an equivalent series resistance,
f_2, f_1 = frequency limits of filter.

For room temperature with a very sharply tuned circuit so $f_2-f_1=1$,

$$V_{\min} \geqq 10^{-10}(R+R_0)^{\frac{1}{2}}. \tag{27a}$$

The values in the fourth column of Table V are calculated from Eq. (27a), with $R_0=10^4$ ohms. $G_{\min}$ was obtained from the relation

$$G_{\min}=V_{\min}/z. \tag{28}$$

It will be noted from the table that on this basis, both the a.c. thermocouple and the a.c. metallic bolometer make a very poor showing.

A third criterion is applicable where the available radiant power is limited, as by the slits and optical system of a monochromator. In such cases the important quantity is the minimum detectable radiant power $\Phi_{\min}$ rather than the minimum detectable irradiation $G_{\min}$. With thermocouples and bolometers, best results under these conditions appear to be obtained by forming an image of the output slit on the target, which is made as small as feasible. In the calculation of Table V, the targets of the thermocouples and bolometers are 1 mm² in area. The minimum area of the dielectric-bolometer target is taken as 1 cm², since smaller targets will generally result in too high an impedance. The values in the seventh column are calculated from the relation

$$\Phi_{\min}=G_{\min}A. \tag{29}$$

It will be noted that the photoelectric devices rank highest on all three bases, though it should be remembered that these values apply only at the wave-length of maximum response. The vacuum thermocouple with galvanometer has the lowest $\Phi_{\min}$ obtained experimentally on any nonselective device, a result in agreement with the conclusions of Cartwright.[26, 27]

From Table V one concludes that the dielectric bolometer is not at its best in spectroradiometry, though pushed to its theoretical limit it would approach the vacuum thermocouple. When immersed in a uniform radiation field, however, where its large target can be used to advantage, the dielectric bolometer compares favorably with all devices except the photoelectric. The dielectric bolometer compares even more favorably in technical measurements where the values of irradiation are large enough so that no limitation is imposed by Brownian motion. Here the voltage obtained per unit irradiation is greater than for any other nonselective radiometer, while the simplicity of operation, ruggedness, and freedom from spurious effects are also of importance.

10. Possible Limitations

There remains the consideration of various disturbing effects which may limit the accuracy of measurements made with the dielectric bolometer. The following possible sources of trouble may be listed:

(a) Variations in ambient temperature
(b) Variations in supply voltage
(c) Variations in frequency
(d) Changes in the properties of the dielectric ("fatigue," "aging," etc.)
(e) Vibration
(f) Electromagnetic disturbances
(g) Thermal e.m.f.

The sensitivity of any radiometer whose operation depends on the heating of a target is affected by changes in the ambient temperature. In the vacuum thermocouple and the metallic bolometer, the sensitivity varies inversely as the third power of the absolute temperature T_a. In the dielectric bolometer, the variation is complicated by the nature of the a.c. loss curve, which may either increase or decrease the ordinary third-power variation. In the worst case, where Eq. (25) holds, the sensitivity of the device varies as T_a^{-5} instead of T_a^{-3}. A temperature rise of from 20°C to 30°C, which is probably as great a variation in room temperature as would be encountered in the laboratory, reduces the sensitivity to 90 percent and to 85 percent of its 20° value, for T_a^{-3} and T_a^{-5}, respectively. Evidently for accurate work some arrangement must be made to take care of this temperature effect—by frequent calibration, by temperature control, or by an automatically compensating element in the bridge.

A consideration of the available data on the effect of voltage and frequency on the a.c. properties of dielectrics indicates that any ordinary variations that might occur in practice would not affect the dial settings of the bridge. Variations in amplifier gain likewise have no effect on the dial settings. There seems to be no reason why well-constructed bolometer cells

should be appreciably affected by vibration, and our experience with the Pliofilm cells shows them to be decidedly less microphonic than the ordinary vacuum tubes used in the amplifier. Because of the blocking condenser used in the amplifier input, thermal electromotive forces can have no effect. Electromagnetic pick-up from other electric circuits may be troublesome, but no more so than in many other sensitive electrical measuring devices.

There remains the possibility of erratic behavior of the dielectric used in the bolometer cells. The two words "dielectric" and "erratic" are only too closely associated in the minds of those who have dealt extensively with insulating materials. It is believed, however, that most of the anomalous results obtained in commercial tests are caused by changes in humidity, poor contact between electrodes and dielectric, chemical effects at the electrodes, or space-charge effects in the material. With the bolometer elements operated in a vacuum and with permanent electrodes and a.c. of sufficiently high frequency to eliminate space-charge effects, there seems to be a very good chance of obtaining dielectric bolometers that are as dependable as any other radiometers.

Summary

Previous bolometers have been limited in sensitivity (volts per watt/cm²) by

(a) Low temperature coefficient obtainable with metals
(b) Low resistance.

The paper describes a new form of bolometer in which the metallic elements are replaced by plates of insulating material and in which both of the above limitations are removed.

In the first part of the paper, the theory of the dielectric bolometer is developed and the properties of some of the available insulating materials are considered. Calculated values of sensitivity z for these materials range from 2 to 135 volts per watt/cm² at room temperature, with 100 volts, 1000~ applied to the dielectric. The theoretical limit, however, is shown to be of the order of 20,000; while a calculated value for glass at 60~ and 100°C is over 9,000.

Experimental bolometers were constructed with both Cellophane and Pliofilm as dielectrics. An experimental Pliofilm cell gave $z=12.5$ with 8.5 volts, corresponding to $z=148$ for 100 volts. These values are much higher than anything obtained with metallic bolometers or with radiation thermocouples, where z is at best only a few tenths. The bolometers were found to give reproducible results, were easily operated, and were free from trouble caused by vibration. A comparison with other types of radiometers indicates that the dielectric bolometer is not particularly well fitted to spectroradiometric measurements but should be most useful where its large target can be used advantageously and where its ruggedness and portability are of importance.

It is a pleasure to acknowledge the helpful suggestions given by Professor R. D. Bennett, Dr. C. H. Cartwright, Professor T. S. Gray, Professor W. M. Hall, Professor A. C. Hardy, Professor E. A. Hauser, and Dr. G. B. Hoadley.

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Selected Ordinates for Luminosity Computations *

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Wave-lengths have been calculated for use in the thirty selected ordinate method for computing the visual reflectance or transmittance of colored materials illuminated with radiation from black bodies at the temperatures: 2000°K, 2200°K, 2360°K, 2600°K, 2848°K, 3000°K, 3200°K, and 4000°K. Each ordinate of any one of these sets is displaced by an almost constant wave-length interval from the corresponding ordinate of any other set. The average value of this displacement is given very accurately ($\pm 0.10 m\mu$) by the formula:

$$\Delta\lambda = 7.20 \times 10^4 (1/T_1 - 1/T_2) m\mu,$$

where the two sets of wave-lengths correspond to black-body illuminants having the Kelvin temperatures, T_1 and T_2, between 2000°K and 4000°K. The numerical constant of the above equation is proportional to the value of the constant ($C_2 = 14{,}330$) adopted for use in Planck's formula. Table XVII of the *Handbook of Colorimetry* gives the wave-lengths of the thirty selected ordinates for the I.C.I. illuminant "A" (2848°K) and the I.C.I. "visibility" function, $\bar{y}$. These wave-lengths increased by the constant given by the following formula, can be used for any black-body illuminant having the temperature T between 2000°K and 4000°K:

$$\Delta\lambda = (7.20 \times 10^4 / T) - 25.28 m\mu.$$

A. C. Hardy has suggested the use of a transparent plate ruled along the wave-lengths of the selected ordinates for 2848°K when placed over a spectrophotometric curve drawn on some standard size graph paper. Such a plate, displaced toward longer wave-lengths by the amount given by the above formula will serve for the selection of the ordinates for any temperature T between 2000°K and 4000°K. The resulting wave-lengths will have an average error of less than $0.5 m\mu$, and the largest error, occurring at the shortest wave-length, will not be greater than $3.0 m\mu$. Errors exceeding $1 m\mu$ occur only for the first one and the last two ordinates, and only within a few hundred degrees of the limits of the temperature range. The effects of such errors are quite negligible for the types of spectrophotometric curves commonly encountered. The selected ordinates for the I.C.I. $\bar{x}$ and $\bar{z}$ functions for general colorimetric computations do not conform with sufficient accuracy to any law of the above form to justify the labor of evaluation of the corresponding constants.

THE selected ordinate method[1] for the computation of visual reflectance or transmittance of colored materials is by far the most rapid and convenient of the accurate methods available. The most serious objection to this method is that the labor of determining the wave-lengths of the selected ordinates is considerable, and must be repeated for each illuminant. Prior to this publication, these data[2] have been available only for the three I.C.I. illuminants, "A," "B," and "C." Problems are frequently encountered in which black-body illuminants at temperatures other than 2848°K (I.C.I. illuminant "A") are appropriate. In connection with a series of such problems, the wave-lengths for the selected ordinates for black bodies at 2000°K, 2200°K, 2360°K, 2600°K, 3000°K, 3200°K, 4000°K have been computed.[3]

The almost uniform displacement of all of the wave-lengths for one temperature from the corresponding wave-lengths for any other temperature was noticed early in the work, and this fact has been used to eliminate the necessity for any further determinations of sets of wave-lengths for selected ordinates for luminosity calculations with black-body illuminants in this range of temperatures.

Table I lists, under the heading, λ, the computed wave-lengths for the selected ordinates for the black-body illuminants mentioned above, together with the data for 2848°K.[4] The difference between each of these wave-lengths and the corresponding value for 2848°K is listed under the heading, $\Delta\lambda$. The average value of $\Delta\lambda$ is shown at the bottom of each difference column, and is plotted as a function of $1/T$ in Fig. 1. The difference between the individual values of $\Delta\lambda$ and the average value for each temperature is listed under the heading, "Error." These values are the errors which would be introduced if the

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[1] *Handbook of Colorimetry* (Technology Press, Mass. Inst. of Technology, Cambridge, Mass., 1936) Chapter VI.

[2] Reference 1, Tables XVII, XVIII, and XIX, pp. 50–51.

[3] Reference 1, Chapter VI, section 30, p. 49.

[4] Reference 1, Table XVII, p. 50.

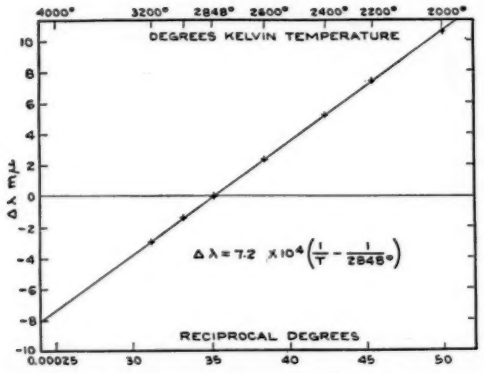

FIG. 1. The average displacement, $\Delta\lambda$, of the selected ordinates for luminosity computations with a black-body at temperature, T, from the corresponding ordinates for 2848°K, represented as a linear function of reciprocal Kelvin temperature.

wave-lengths for the selected ordinates were computed by adding the average value of $\Delta\lambda$ to the published values for 2848°K. These errors are plotted as functions of the number of the ordinate, for the several temperatures, in Fig. 2, and as functions of temperature for the first, fifteenth, and thirtieth ordinates, in Fig. 3.

It is evident from Table I and from Figs. 2 and 3 that the errors introduced by this simplified computation will be less than three millimicrons in the extreme case (first ordinate for 4000°K) and less than one millimicron for all except the first and last two ordinates, near the limits of the temperature range. The average values of the wave-length errors are shown at the bottoms of the "Error" columns in Table I, and are plotted in Fig. 4. This average error is less than 0.5 mμ for all temperatures within the range from 2000°K to 4000°K. These small errors of wave-length can cause appreciable errors in only those few ordinates which correspond to steep portions of the spectrophotometric curve for which the visual reflectance or transmittance is being evaluated. Since the computed visual reflectance or trans-

TABLE I. *Selected ordinates for luminosity (black-body illuminants).*

2000°K			2200°K			2360°K			2600°K		
λ	Δλ	Error	λ	Δλ	Error	λ	Δλ	Error	λ	Δλ	Error
500.29	12.47	+1.88	496.79	8.98	+1.57	494.16	6.35	+1.13	490.65	2.84	+0.46
518.01	10.26	−0.33	514.88	7.14	−0.27	512.90	5.16	−0.06	509.89	2.15	−0.23
527.29	10.01	−0.58	524.16	6.88	−0.53	522.17	4.89	−0.33	519.37	2.09	−0.29
534.26	10.15	−0.44	531.14	7.03	−0.38	529.03	4.92	−0.30	526.30	2.19	−0.19
540.19	10.40	−0.19	537.02	7.23	−0.18	534.81	5.02	−0.20	532.08	2.29	−0.09
545.43	10.62	+0.03	542.20	7.39	−0.02	539.95	5.14	−0.08	537.16	2.35	−0.03
550.20	10.79	+0.20	546.92	7.52	+0.11	544.64	5.24	+0.02	541.79	2.39	+0.01
554.62	10.92	+0.33	551.30	7.61	+0.20	549.01	5.32	+0.10	546.11	2.42	+0.03
558.79	11.02	+0.43	555.43	7.66	+0.25	553.16	5.40	+0.18	550.20	2.44	+0.06
562.76	11.09	+0.50	559.38	7.72	+0.31	557.10	5.44	+0.22	554.13	2.47	+0.09
566.54	11.10	+0.51	563.19	7.76	+0.35	560.87	5.44	+0.22	557.93	2.50	+0.12
570.21	11.10	+0.51	566.87	7.76	+0.35	564.56	5.45	+0.23	561.60	2.49	+0.11
573.81	11.10	+0.51	570.47	7.76	+0.35	568.17	5.46	+0.24	565.20	2.49	+0.11
577.35	11.09	+0.50	574.01	7.75	+0.34	571.71	5.45	+0.23	568.76	2.50	+0.12
580.84	11.05	+0.46	577.53	7.74	+0.33	575.23	5.44	+0.22	572.29	2.50	+0.12
584.33	11.01	+0.42	581.03	7.71	+0.30	578.75	5.43	+0.21	575.82	2.50	+0.12
587.83	10.96	+0.37	584.56	7.69	+0.28	582.27	5.40	+0.18	579.36	2.49	+0.11
591.38	10.92	+0.33	588.11	7.65	+0.24	585.85	5.39	+0.17	582.93	2.47	+0.09
594.99	10.87	+0.28	591.73	7.61	+0.20	589.50	5.38	+0.16	586.58	2.46	+0.08
598.67	10.78	+0.19	595.46	7.58	+0.17	593.22	5.34	+0.12	590.34	2.46	+0.08
602.45	10.67	+0.08	599.30	7.52	+0.11	597.07	5.29	+0.07	594.22	2.44	+0.06
606.42	10.57	−0.02	603.28	7.43	+0.02	601.10	5.25	+0.03	598.29	2.44	+0.06
610.59	10.44	−0.15	607.45	7.31	−0.10	605.34	5.20	−0.02	602.53	2.39	+0.01
615.00	10.26	−0.33	611.94	7.21	−0.20	609.85	5.12	−0.10	607.09	2.36	−0.02
619.78	10.07	−0.52	616.77	7.07	−0.34	614.73	5.03	−0.19	612.02	2.32	−0.06
625.09	9.87	−0.72	622.10	6.88	−0.53	620.13	4.92	−0.30	617.50	2.28	−0.10
631.22	9.70	−0.89	628.30	6.78	−0.63	626.32	4.80	−0.42	623.77	2.25	−0.13
638.81	9.59	−1.00	635.94	6.72	−0.69	633.97	4.75	−0.47	631.44	2.22	−0.16
649.09	9.37	−1.22	646.24	6.53	−0.88	644.35	4.64	−0.58	641.85	2.14	−0.23
668.22	9.25	−1.34	665.58	6.62	−0.79	663.54	4.58	−0.64	661.15	2.18	−0.29
Average:	10.59	0.51		7.41	0.37		5.22	0.25		2.38	0.12
Formula:	10.72			7.45			5.23			2.41	

mittance is the average of thirty ordinates, only a few of which are subject to any appreciable error, the errors introduced by this convenient method of approximation will be negligible. If corrections to the wave-lengths of some ordinates are considered necessary, they can be computed from Table I or from Figs. 2 or 3.

The average value of $\Delta\lambda$, to be added to the wave-lengths for 2848°K, can be read from Fig. 1 or can be computed from the formula:

$$\Delta\lambda = 7.2\times10^4(1/T - 1/2848°\text{K})\ \text{m}\mu, \quad (1a)$$

$$\Delta\lambda = 7.2\times10^4/T - 25.28\ \text{m}\mu. \quad (1b)$$

In general, the average difference between corresponding wave-lengths for black-body illuminants at temperatures T_1 and T_2 will be given by

$$\Delta\lambda = 7.2\times10^4(1/T_1 - 1/T_2)\ \text{m}\mu. \quad (2)$$

The errors shown in Figs. 2 and 3 are additive

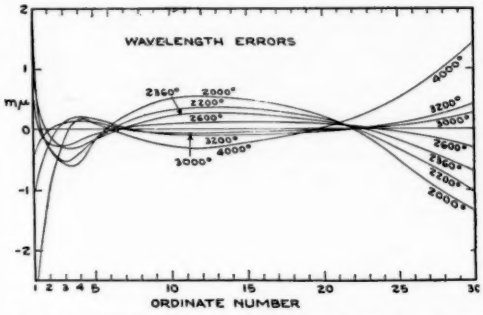

FIG. 2. Wave-length errors of the selected ordinates computed by adding the average value of $\Delta\lambda$ to the wave-lengths of the selected ordinates for 2848°K, for the indicated illuminant temperatures.

when $T_1 < 2848°\text{K} < T_2$, and should be subtracted when $T_1 < T_2 < 2848°\text{K}$, or when $2848°\text{K} < T_1 < T_2$. The formula for the average value of $\Delta\lambda$ applies for temperatures considerably beyond the limits, 2000°K and 4000°K, but the errors in-

TABLE I.—*Continued.*

2848°K	3000°K			3200°K			4000°K		
λ	λ	Δλ	Error	λ	Δλ	Error	λ	Δλ	Error
487.82	486.04	−1.77	−0.41	483.91	−3.90	−0.99	478.03	−9.78	−2.85
507.75	506.38	−1.36	0.00	504.64	−3.11	−0.20	499.79	−7.95	−1.02
517.28	516.02	−1.26	+0.10	514.47	−2.81	+0.10	510.17	−7.10	−0.17
524.11	522.91	−1.20	+0.16	521.37	−2.74	+0.17	517.34	−6.77	+0.16
529.79	528.50	−1.29	+0.10	527.04	−2.75	+0.16	522.96	−6.83	+0.10
534.81	533.50	−1.31	+0.05	532.02	−2.79	+0.12	527.90	−6.91	+0.02
539.41	538.04	−1.37	−0.01	536.53	−2.87	+0.04	532.40	−7.00	−0.07
543.70	542.32	−1.38	−0.02	540.77	−2.93	−0.02	536.58	−7.11	−0.18
547.77	546.36	−1.40	−0.04	544.80	−2.96	−0.05	540.57	−7.19	−0.26
551.67	550.24	−1.42	−0.06	548.69	−2.98	−0.07	544.43	−7.23	−0.30
555.44	554.01	−1.43	−0.07	552.46	−2.98	−0.07	548.19	−7.25	−0.32
559.11	557.69	−1.42	−0.06	556.10	−3.01	−0.10	551.85	−7.26	−0.33
562.71	561.28	−1.43	−0.07	559.70	−3.01	−0.10	555.47	−7.24	−0.31
566.26	564.85	−1.41	−0.05	563.27	−2.99	−0.08	559.05	−7.21	−0.28
569.79	568.38	−1.41	−0.05	566.79	−3.00	−0.09	562.62	−7.17	−0.24
573.32	571.91	−1.41	−0.05	570.32	−3.00	−0.09	566.20	−7.12	−0.19
576.87	575.46	−1.41	−0.05	573.88	−2.99	−0.08	569.80	−7.07	−0.14
580.46	579.07	−1.39	−0.03	577.46	−3.00	−0.09	573.44	−7.02	−0.09
584.12	582.72	−1.40	−0.04	581.15	−2.97	−0.06	577.19	−6.93	0.00
587.89	586.50	−1.38	−0.02	584.93	−2.95	−0.04	580.98	−6.90	+0.03
591.78	590.41	−1.37	−0.01	588.85	−2.93	−0.02	584.95	−6.83	+0.10
595.85	594.49	−1.36	0.00	592.92	−2.93	−0.02	589.09	−6.74	+0.19
600.15	598.80	−1.34	+0.02	597.24	−2.90	+0.01	593.46	−6.68	+0.25
604.73	603.40	−1.33	+0.03	601.87	−2.86	+0.05	598.15	−6.58	+0.35
609.71	608.40	−1.30	+0.06	606.90	−2.80	+0.11	603.26	−6.44	+0.49
615.22	613.96	−1.25	+0.11	612.45	−2.77	+0.14	608.95	−6.26	+0.67
621.52	620.31	−1.21	+0.15	618.88	−2.65	+0.23	615.47	−6.05	+0.88
629.22	628.03	−1.19	+0.17	626.62	−2.60	+0.31	623.30	−5.92	+1.01
639.72	638.55	−1.16	+0.20	637.14	−2.58	+0.33	634.01	−5.70	+1.23
658.97	657.85	−1.11	+0.25	656.42	−2.55	+0.36	653.43	−5.53	+1.40
Average:		−1.36	0.08		−2.91	0.14		−6.93	0.46
Formula:		−1.28			−2.78			−6.74	

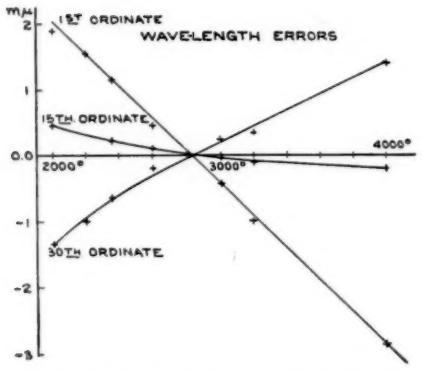

FIG. 3. Wave-length errors of the first, fifteenth, and thirtieth selected ordinates computed by adding the average value of $\Delta\lambda$ to the wave-lengths of the selected ordinates for 2848°K, shown as functions of illuminant temperature.

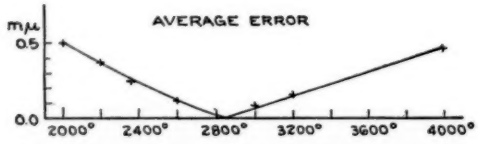

FIG. 4. Average wave-length errors (regardless of sign) of the selected ordinates computed by adding the average value of $\Delta\lambda$ to the wave-lengths for 2848°K, shown as functions of illuminant temperature.

crease rapidly as shown by the trends of the curves in Figs. 2, 3, and 4.

Hardy[5] has suggested that the selected ordinates for 2848°K can be conveniently indicated by parallel lines ruled on the under side of a transparent plate, and which are at the correct positions along the wave-length scale when the plate is placed over any spectrophotometric curve drawn on standard size graph paper. Such a plate can be used to select the ordinates for any other black-body illuminant between 2000°K and 4000°K, if it is simply displaced along the wave-length scale the amount indicated by Fig. 1 or by Eq. (1b).

Table II gives the wave-lengths for the selected ordinates for the I.C.I. color mixture functions, $\bar{x}$ and $\bar{z}$, with the black-body illuminant at 2360°K. The columns, $\Delta\lambda$, show the differences between the listed values and the corresponding values for 2848°K. The "Error" columns show the errors that would be introduced if these wave-lengths were computed by adding the average value of the differences, $\Delta\lambda$, to the values for 2848°K. These errors are much more serious than were those for the luminosity function. Consequently, this method of computing the wave-lengths of the selected ordinates for the color mixture functions, $\bar{x}$ and $\bar{z}$, cannot be recommended.

TABLE II. *Selected ordinate wave-lengths for 2360°K.*

X			*Z*		
λ	Δλ	Error	λ	Δλ	Error
458.8	14.9	+9.5	418.4	2.0	−1.2
540.1	23.2	+17.8	426.8	2.0	−1.2
553.5	9.5	+4.1	431.4	2.0	−1.2
561.7	7.6	+2.2	435.1	2.2	−1.0
567.9	6.5	+1.1	438.3	2.3	−0.9
573.1	6.0	+0.6	441.2	2.5	−0.7
577.5	5.5	+0.1	443.9	2.6	−0.6
581.4	5.1	−0.3	446.4	2.7	−0.5
585.1	4.9	−0.5	448.8	2.8	−0.4
588.6	4.8	−0.6	451.1	2.8	−0.4
591.8	4.6	−0.8	453.4	2.9	−0.3
594.8	4.4	−1.0	455.6	3.0	−0.2
597.7	4.2	−1.2	457.7	3.0	−0.2
600.6	4.1	−1.3	459.8	3.1	−0.1
603.4	4.0	−1.4	461.8	3.0	−0.2
606.2	3.9	−1.5	463.8	3.0	−0.2
609.0	3.8	−1.6	465.8	2.9	−0.3
611.7	3.7	−1.7	467.9	3.0	−0.2
614.5	3.6	−1.8	470.2	3.1	−0.1
617.4	3.6	−1.8	472.5	3.3	+0.1
620.4	3.5	−1.9	474.9	3.3	+0.1
623.6	3.6	−1.8	477.5	3.4	+0.2
627.0	3.7	−1.7	480.5	3.7	+0.5
630.5	3.6	−1.8	483.7	3.8	+0.6
634.2	3.4	−2.0	487.3	3.9	+0.7
638.6	3.3	−2.1	491.7	4.2	+1.0
643.8	3.4	−2.0	497.1	4.4	+1.2
650.4	3.5	−1.9	504.0	4.7	+1.5
659.2	3.5	−1.9	513.0	4.6	+1.4
677.1	3.7	−1.7	532.0	5.3	+2.1
Average:	5.4		Average:	3.2	

Factors (30 ordinates)	0.03929	0.00772
Factors (10 ordinates)	0.11786	0.02316

Illuminant point: $x=0.4890$, $y=0.4149$

[5] A. C. Hardy, "A New Recording Spectrophotometer," J. Opt. Soc. Am. **25**, 311 (1935).

Note on the Photon Emission of a Black Body

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The total number of photons emitted by a black body in thermodynamical equilibrium is found to be $1.5\times10^{11}\times T^3$ photons per sec. per cm^2. The total number of photons emitted by the sun in a period of about 20,000 years equals the total number of protons and neutrons contained in the sun.

THE number of photons which are emitted per second per sq. cm by a black body in thermodynamical equilibrium at a given temperature may be computed on the basis of Planck's radiation law. If this number be denoted by n and the emissive power be spectrally resolved according to the relation

$$K=\int_0^\infty K_\nu d\nu \tag{1}$$

we find $$n=\int_0^\infty \frac{K_\nu}{h\nu}d\nu \tag{2}$$

where $$K_\nu=((2\pi h\nu^3)/c^2)(e^{(h\nu)/(kT)}-1)^{-1} \tag{3}$$

and T, k, and c denote the absolute temperature, Boltzmann's constant, and the velocity of light, respectively. If, for brevity's sake, we put

$$1+2^{-3}+3^{-3}+\cdots=1.20206=\beta \tag{4}$$

we find

$$n=(4\pi\beta k^3T^3)/(h^3c^2)=1.5\times10^{11}\times T^3. \tag{5}$$

Since the radius of the sun is about 7.0×10^{10} cm, the number of photons emitted per sec. by the sun amounts to $9.2\times10^{33}\times T^3$, and hence by assuming the surface temperature of the sun to be about 6000° abs., we find the number of photons to be

$$Z=2\times10^{45}. \tag{6}$$

Since the sun has a mass of about 2×10^{33} grams, the total number of protons and neutrons contained in the sun must be assumed to be about 1.2×10^{57}. An equal number of photons is emitted by the sun in about 6×10^{11} sec. or in a period of approximately only 20,000 years.

We must therefore either assume that each primordial particle contained in the sun experiences, at least in intervals of about 20,000 years, some reaction leading to the emission of a photon, or we must assume that extremely "hard" primary photons produced in the interior of the sun split into a variety of softer photons on the way to the surface of the sun, or perhaps we must combine both assumptions.

The question might also be of some interest as to what the distance of a star of the absolute luminosity and the surface temperature of the sun should be in order that it would send only a single photon per sec. through a sq. cm area at the surface of the earth. From Eq. (6) this distance is found to be $[Z/(4\pi)]^{\frac{1}{2}}$ or approximately 1.3×10^{22} cm, and thus about 13,000 light years.

The average energy of a photon emitted in thermodynamical equilibrium is a/n, where a, the constant of Stefan's law, is

$$a=(2\pi^5k^4T^4)/(15h^3c^2). \tag{7}$$

The mean energy of a photon is thus found to be

$$a/n=(\pi^4kT)/(30\beta)=2.701kT. \tag{8}$$

The wave-length corresponding to this average value of the photon energy is 5.514 times Wien's wave-length, for which the specific emissive power is a maximum, and amounts to about 26,000A for the solar radiation.

Annual Report of the Secretary of the Optical Society of America for the Calendar Year 1937

Membership Record

January 1, 1937 to January 1, 1938

	Honorary	Regular Annual Payment	Regular Life Payment	Associate Individual Annual Payment	Associate Corporation	Total
Membership as of record in the books of the Secretary and Treasurer 1–1–37	4	399	6	199	9	617
Gains during 1937						
By election during 1937 as of 1–1–37	0	0	0	26+	0	26+
Losses during 1937						
By death	1−	2−	0	0	0	3−
By resignation as of 1–1–37	0	7−	0	7−	0	14−
By separation for nonpayment of dues	0	5−	0	5−	0	10−
Membership as of record 12–31–37	3	385	6	213	9	616
Gains between 12–31–37 and 1–1–38						
By election during 1937 as of 1–1–38	0	0	0	15+	0	15+
By transfer 10–14–37 as of 1–1–38	0	26+	0	0	0	26+
Losses between 12–31–37 and 1–1–38						
By transfer 10–14–37 as of 1–1–38	0	0	0	26−	0	26−
By resignation during 1937 as of 12–31–37	0	5−	0	3−	0	8−
Membership as of record 1–1–38	3	406	6	199	9	623

Elected during 1937 as of January 1, 1937

Gardner R. Alden (1072 A)
Calvin B. Coulter (1085 R)
L. W. Doolan, Jr. (1083 A)
Beverly Dudley (1084 A)
Paul C. Foote (1105 A)
John M. Gallagher, Jr. (1081 A)
Parker Hamilton (1092 A)
Robt. E. Hopkins (1091 A)
Elmer Hutchisson (1088 R)
Richard F. Jarrell (1098 A)
Grandville R. Jones (1079 A)
C. A. Robert Lundin (1077 R)
Norman Macbeth (1087 A)
William Mayer (1076 R)
Kenneth N. Ogle (1075 R)
Roy A. Palmer (1086 R)
Allan E. Parker (1099 R)
Gerald M. Rassweiler (1074 R)
Ferdinand Schurman (1094 A)
Paul W. Stevens (1080 A)
Frank J. Studer (1089 R)
Jack H. Waggoner (1090 A)
John F. Walton (1073 R)
Milton L. Weiss (1078 A)
John M. Wells (1100 R)
Richard H. Zinszer (1082 A)

Elected during 1937 as of January 1, 1938

Frank Avignone, Jr. (1108 A)
H. S. Denmark (1109 A)
Forrest L. Dimmick (1095 R)
Isidore S. Finkelstein (1107 A)
Edward F. Flint, Jr. (1112 A)
Walter W. Graeper (1111 A)
Cedric E. Hesthal (1101 R)
Frederick W. Jobe (1102 A)
Henry C. Ketcham (1096 A)
Victor R. Knop (1104 A)
Robert H. Osborn (1097 A)
Carl F. J. Overhage (1093 A)
Seymour Rosin (1103 A)
Irvin Stewart (1110 A)
Arthur F. Turner (1106 A)

Elected to Regular Membership by Action of the Board of Directors, October 14, 1937, effective January 1, 1938

Ernest H. Anthes (951 R)
Ellsworth M. Brockway (937 R)
Victor E. Carbonara (895 R)
G. Howard Carragan (1060 R)
Olindo O. Ceccarini (583 R)
Calvin B. Coulter (1085 R)
Forrest L. Dimmick (1095 R)
R. H. Draeger (1063 R)
Ralph W. Duncan (915 R)
Roger S. Estey (848 R)
Edward E. Free (1065 R)
Martin Grabau (940 R)
Cedric E. Hesthal (1101 R)
Elmer Hutchisson (1088 R)
J. F. Leventhal (1035 R)
C. A. Robert Lundin (1077 R)
Wm. Orland Lytle (1068 R)
William Mayer (1076 R)
Julius F. Neumueller (1030 R)
Kenneth N. Ogle (1075 R)
Roy A. Palmer (1086 R)
Allan E. Parker (1099 R)
Gerald M. Rassweiler (1074 R)
Frank J. Studer (1089 R)
Karl S. Weaver (1069 R)
John M. Wells (1100 R)

Deaths during 1937

Frederic E. Ives (268 R) 5–27–37
Edward L. Nichols (60 H) 11–10–37
Elihu Thomson (280 R) 3–13–37

Resignations during 1937 as of January 1, 1937

Edward L. Bowles (450 A)
Mabel E. Brown (611 A)
Harry B. Cuthbertson (1021 A)
T. L. DeBruin (582 A)
A. Joffe (470 R)
Frank C. McDonald (723 A)
Daniel Smith (1037 A)
Scott W. Smith (896 A)
F. M. Walters, Jr. (160 R)
Frank Wenner (739 A)
Reynold K. Young (876 R)

Resignations during 1937 as of December 31, 1937

Barnett Cohen (690 R)
John B. Johnson (407 R)
Charles T. Knipp (534 R)
Henry Laurens (220 R)
Oliver C. Lester (843 A)
Herman E. Seeman (778 R)
G. M. Shrum (537 R)
William O. Smith (586 A)
George H. Stickney (222 R)
David L. Webster (527 R)
Walter Wrigley (960 A)

(*Signed*) L. B. TUCKERMAN, *Secretary*

Annual Report of the Treasurer of the Optical Society of America for the Fiscal Year 1937

CASH TRANSACTIONS JANUARY 1, 1937 TO DECEMBER 31, 1937

Receipts

Dues	$4292.10	
Dues in advance	2330.15	
Interest on bonds	942.25	
Interest on bank balance	101.24	
Interest on Frederic Ives Medal Fund bonds	15.00	
Interest on Adolph Lomb Memorial Fund bonds	65.00	
Bank interest on Frederic Ives Medal Fund	1.56	
Bank interest on Adolph Lomb Memorial Fund	5.15	
Bank interest on Local Committee Fund	2.85	
Unbudgeted Income	14.70	
		$7770.00

Disbursements

Secretary's Expense	$ 478.52	
Secretary's Travel Expense	101.25	
Treasurer's Expense	115.81	
Journal Publication Expense	5562.38	
Journal Editorial Expense	1225.00	
Contingencies	39.30	
Specially appropriated expenditures		
Contribution to U. S. Committee I. C. I.	100.00	
Purchase of Addressograph	642.08	
Frederic Ives Medal Purchase	18.14	
Adolph Lomb Memorial Fund purchase expense	1.25	
		$8283.73
Excess Disbursements over Receipts for period January 1 to December 31, 1937		$ 513.73
Cash in Bank January 1, 1937	$6872.38	
Balance Interest A. Lomb Fund, Jan. 1, 1937	207.25	
Balance Interest Ives Medal Fund, Jan. 1, 1937	82.02	
Local Committee Fund, Jan. 1, 1937	141.54	
		$7303.19
Total cash in bank, December 31, 1937		$6789.46

Frederic Ives Medal Fund

Income

Principal	$ 500.00	
Cash interest balance, Jan. 1, 1937	82.02	
Bond interest accrued during 1937	15.00	
Bank interest on cash balance	1.56	
		$ 598.58

Disbursements

Medal, case and incidental expense on medal awarded to Herbert E. Ives	18.14	
		18.14
Balance—December 31, 1937		$ 580.44

Note: The principal $500.00 is invested in U. S. Treasury Certificates. They are included in the bonds reported in the accompanying investment statement.

Adolph Lomb Memorial Fund

Income

Invested Principal	$1919.06	
Uninvested principal, bond and bank interest, January 1, 1937	207.25	
Bond interest December 31, 1937	65.00	
Bank interest December 31, 1937	5.15	
		$2196.46

Disbursements

Bonds bought in 1936	$1919.06	
Trust Company charge	1.25	
		1920.31
Balance interest and uninvested capital, December 31, 1937		$ 276.15

Note: The invested principal is covered by Dominion of Canada 3¼% bonds bought during 1936. They are included in the bonds reported in the accompanying investment statement.

Local Committee Fund

Income

Balance, January 1, 1937	$ 141.54
Bank interest, December 31, 1937	2.85
	$ 144.39

Disbursements

	0.00
Balance—December 31, 1937	$ 144.39

Securities held by Security Trust Company of Rochester as Agent for Optical Society of America

	Cost	Dec. 2, 1937 Market Value	Dividends During 1937	Rate of Return Against Cost
Bonds				
$7000 Atchison Topeka & Santa Fe—4%	$ 6916.25	$ 7595.00	$ 280.00	4.05%
$2000 Dominion of Canada—3¼%	1918.70	1997.50	65.00	3.39%
$5600 U. S. Treasury Certificates—3¼%	5625.67	5952.80	182.00	3.23%
$8000 U. S. Treasury Certificates—3%	8261.30	8343.20	240.00	2.91%
Total Bonds	$22721.92	$23888.50	$ 767.00	3.39%
Stocks				
10 shares Eastman Kodak Common	$ 1731.40	$ 1510.00	$ 77.50	4.48%
40 shares General Electric Common	1871.35	1715.00	78.00	4.17%
35 shares Kennecott Copper Common	1729.53	1264.38	99.75	5.77%
Total Stocks	$ 5332.28	$ 4489.38	$ 255.25	4.80%
Totals and Average Return	$28054.20	$28377.88	$1022.25	3.64%

March 22, 1938

The undersigned, having been duly appointed to audit the books of the Treasurer of the Optical Society of America, Inc., from January 1, 1937 to December 31, 1937, hereby certify that the Treasurer's report of receipts, disbursements, and investments is correct as shown by records, vouchers, bank statements, etc.

(*Signed*) Carl L. Bausch, C. W. Frederick, *Auditors*

Report of the Editor of Publications

STATEMENT OF OPERATIONS OF THE JOURNAL OF THE OPTICAL SOCIETY OF AMERICA FOR THE YEAR ENDING DECEMBER 31, 1937

A. In Account with American Institute of Physics Inc.

Income			
Subscriptions		$3,959.19	
Net Income from published articles (Schedule 1)		1,049.63	
Net Income from single copies (Schedule 1)		321.59	
Miscellaneous		14.98	$5,345.39
Expenses			
Printing and mailing		$4,688.18	
Editorial expenses (A. I. P. office)			
Salaries	$446.16		
Office expenses	121.29	567.45	
Business management expenses			
Salaries	510.15		
Office expenses	132.72	,642.87	
Miscellaneous		100.73	
Total expenses			$5,999.23
Excess of expense over income			$ 653.84
Service charge of American Institute of Physics			1,037.53
Net cost of editorial mechanics, printing and distributing journal			$1,691.37

B. Expenses in editorial office (Ithaca)

Salaries	$1,000.00	
Miscellaneous	96.57	
Total in Ithaca office		$1,096.57
Total net cost of journal for 1937		$2,787.94

Number of pages published	1937	1936	1935	1934
Text and index	437	460	426	355
Covers and front matter	72	82	78	80
Total	509	542	504	435

Cumulative Index of the Journal of the Optical Society of America

Income		
Sales of Cumulative Index		$221.09
Expense		
Handling, mailing and advertising	$151.53	
15% Service charge	22.73	174.26
Net income		$ 46.83

SCHEDULE 1

Net Income from Published Articles

Income		
"Per Page" publication charges	$1,170.00	
Sales of reprints	698.71	$1,868.71
Expense (in A. I. P. office)		
Manufacture and mailing of reprints	$ 714.17	
Business management salaries	83.39	
Business management office expense	21.52	819.08
Net Income from published articles		$1,049.63

Net Income from Sale of Single Copies

Income—Sales of Single Copies		$ 420.13
Expense (in A. I. P. office)		
Business management salaries	$ 78.34	
Business management office expense	20.20	98.54
Net Income from sales of single copies		$ 321.59

(*Signed*) F. K. RICHTMYER, *Editor*

Proceedings of the New York Meeting of the Optical Society of America

FEBRUARY 25 AND 26, 1938

JOINT MEETING WITH THE AMERICAN PHYSICAL SOCIETY

THE joint meeting of the Optical Society of America and the American Physical Society was held in New York City, February 25 and 26, 1938.

JOINT SYMPOSIUM ON THE OPTICAL PROPERTIES OF METALS

A feature of the meeting was the joint symposium of the two societies on Friday morning, February 25, devoted to invited papers on The Optical Properties of Metals. The following papers were presented:

1. **Some Simple Concepts in the Quantum Theory of Metals.** L. A. DUBRIDGE, University of Rochester.
2. **The Experimental Determination of Optical Constants of Metals—Methods and Results.** J. B. NATHANSON, Carnegie Institute of Technology.
3. **Derivation of Properties of Electrons in Metals from Optical Constants.** H. A. BETHE, Cornell University.

ADOLPH LOMB MEMORIAL LECTURE

The principal feature of the meeting was the first Adolph Lomb Memorial Lecture held at the American Museum of Natural History Friday evening, February 25. President Gibbs introduced Professor J. P. C. Southall who spoke briefly of the debt of the Society to Adolph Lomb and the establishment of the Adolph Lomb Memorial Fund. He then introduced Dr. Harlow Shapley, director of the Harvard University Observatory, who spoke on "The Debt of the World to Optical Science."

CONTRIBUTED PAPERS

Simultaneous sessions of the American Physical Society and the Optical Society of America for the reading of contributed papers were held on Friday afternoon, February 25, and Saturday morning, February 26. The authors' abstracts of the papers presented before the Optical Society of America are appended.

INTERSOCIETY COLOR COUNCIL

The members of the Society were invited to attend the technical sessions of the Intersociety Color Council held on Thursday at the Electrical Testing Laboratories. Many members availed themselves of this opportunity.

MEETING OF THE BOARD OF DIRECTORS

A meeting of the Board of Directors was held Thursday morning and evening, February 24. Minutes of this meeting are appended.

L. B. TUCKERMAN, *Secretary*

Minutes of the Eleventh Meeting of the Directors of the Optical Society of America, Incorporated

NEW YORK, N. Y., FEBRUARY 24, 1938

BESIDES the transaction of routine business, the following items of general interest to members of the Society were taken up:

1. Announcements by Secretary

Result of letter ballots:

1. Approval of appointment of A. G. Worthing as member-at-large of the Board of Directors vice K. S. Gibson, resigned, to serve until the annual meeting in October 1938.

2. Appointments of Committees and Representatives

a. Committee on Meetings and Programs

A. G. WORTHING, *Chairman*

E. Q. ADAMS — R. KINGSLAKE
A. W. KENNEY — DOROTHY NICKERSON
FRANCES G. WICK

b. Committee on Finance

H. F. KURTZ, *Chairman*

C. L. BAUSCH — L. A. JONES
L. A. DUBRIDGE — F. K. RICHTMYER
G. R. HARRISON — J. P. C. SOUTHALL

c. Committee on Honorary Members
H. E. IVES, *Chairman*
K. S. GIBSON G. R. HARRISON

d. Committee to Consider Publication of Colorimetry Report
F. K. RICHTMYER, *Chairman*
H. A. BARTON L. A. JONES

e. Auditors of Treasurer's Accounts
C. L. BAUSCH C. W. FREDERICK

f. Membership Committee
W. B. RAYTON, *Chairman*
A. H. BENNETT G. W. MOFFITT
L. E. DODD DOROTHY NICKERSON
W. E. FORSYTHE L. W. TILTON
A. C. HARDY T. R. WILKINS

g. Colorimetry Committee
L. A. JONES, *Chairman*
E. Q. ADAMS D. B. JUDD
CHAS. BITTINGER E. M. LOWRY
E. C. CRITTENDEN M. LUCKIESH
C. Z. DRAVES D. L. MACADAM
J. W. FORREST S. M. NEWHALL
C. E. FOSS DOROTHY NICKERSON
H. P. GAGE BRIAN O'BRIEN
K. S. GIBSON M. R. PAUL
I. H. GODLOVE LOUISE L. SLOAN
A. C. HARDY W. T. SPRY
W. B. VAN ARSDEL

h. Committee on Transfer from Associate to Regular Membership
W. E. FORSYTHE, *Chairman*
W. B. RAYTON K. S. GIBSON

i. Committee on Editorial Policy of Journal
F. K. RICHTMYER, *Chairman*
A. C. HARDY H. E. IVES
G. R. HARRISON L. A. JONES
L. W. MCKEEHAN

j. Representatives on U. S. National Committee of the International Commission on Illumination
FRANK BENFORD, *Chairman*
L. A. JONES A. C. HARDY

k. Representatives on the Intersociety Color Council
L. A. JONES, *Chairman*, voting delegate
I. H. GODLOVE, voting delegate
D. B. JUDD, voting delegate
and J. W. FORREST A. C. HARDY
H. P. GAGE W. B. VAN ARSDEL

l. Representative and alternate respectively on the Sectional Committee of the American Standards Association on Safety Code for Specifications and Methods of Test for Safety Glass-Z26
E. D. TILLYER H. F. KURTZ

m. Representative on the Motion Picture Sectional Committee of the American Standards Association
A. C. HARDY

3. Nominations of Representatives

a. Representative on the Governing Board of the American Institute of Physics
A. C. HARDY

b. Representative on the Division of Physical Sciences, National Research Council
G. B. PEGRAM

4. The following committees and representatives were appointed:

a. Nominating Committee (to fill vacancy on the Board of Directors, October 1938)
A. C. HARDY, *Chairman*
K. S. GIBSON WM. F. MEGGERS
H. F. KURTZ L. B. TUCKERMAN

b. Committee on Date of Election
A. C. HARDY, *Chairman* L. B. TUCKERMAN

c. Advisory Committee on Progress Reports
R. C. GIBBS, *Chairman*
A. C. HARDY A. G. WORTHING

d. Representatives on the Council of the American Association for the Advancement of Science
For a term ending in February 1939, R. C. GIBBS
For a term ending in February 1940, ELMER HUTCHISSON

e. Representative on the American National Committee on International Congresses on Photography for a term ending October 1941
A. C. HARDY

2. Publication of New Directory

The Editor of Publications was authorized to publish a Directory as a regular issue of the Journal.

3. Annual Meeting

It was voted that the annual meeting of the Society be held at the General Brock Hotel, Niagara Falls, Canada, October 27–29, 1938.

L. B. TUCKERMAN, *Secretary*

Program of Sessions for Contributed Papers

TITLES AND ABSTRACTS OF PAPERS

1. Photoelectrically Balanced Recording Potentiometer.* C. OWEN FAIRCHILD AND V. LAWRENCE PARSEGIAN, *C. J. Tagliabue Mfg. Co., Brooklyn, N. Y.*

The problem of stopping a moving light beam on the edge of a phototube without any tendency to oscillate or "hunt," and with speed and precision has been solved by a new method and a recording potentiometer is rapidly and accurately balanced. By (1) artificially canceling the generated back e.m.f. of the moving galvanometer, thus advancing its return movement from a deflected position, and by (2) delaying the grid potential change of a triode amplifier in the photoelectric circuit, galvanometer lag is counteracted and the potentiometer is balanced with an accuracy of a few hundredths of a percent of the range. The total power input of the instrument is about 35 watts and the contact carriage speed is made as high as 10" in 20 seconds. The fundamentals of the complete instrument are discussed and equations describing the galvanometer movement are included.

Bibliography:
1. Hazen, Jaeger and Brown, Rev. Sci. Inst. **7**, 353 (1936).
2. Whitford and Kron, Rev. Sci. Inst. **8**, 78 (1937).
3. Instruments **8**, 343 (1935).
4. Gulliksen and Vedder, "Industrial Electronics" (John Wiley & Sons, 1935).

* The present paper will appear in full in R. S. I.

2. The Second Spectrum of Vanadium.* WILLIAM F. MEGGERS, *National Bureau of Standards*, AND CHARLOTTE E. MOORE, *Princeton University.*

A partial analysis of the V II spectrum presented by one of us[1] in 1926 classified about 280 lines as combinations of 11 triplet and 9 quintet terms, but further progress was prevented by inadequate data. Last year, a new description of vanadium spark spectra was prepared and the term analysis of the V II spectrum was resumed. At the present time nearly 1200 lines have been explained as combinations of 21 singlet, 37 triplet, and 12 quintet terms. Theoretically, the electron configurations $3d^4$ and $3d^34s$ each give rise to 16 terms. All of these have been found except 2 high singlets of the former and 1 high singlet of the latter. The ground state of the V^+ ion is represented by $(3d^4)^5D_0$ and the spectroscopic ionization potential is approximately 14 volts. Observations of the Zeeman effect by H. D. Babcock have greatly facilitated this analysis of the second spectrum of vanadium.

* The present paper will appear in full in Nat. Bur. Stand. J. Research.
[1] W. F. Meggers, Zeits. f. Physik **33**, 509 (1925); Ibid. **39**, 114 (1926).

3. Radiation Laws Describing the Emission of Photons by Black Bodies. A. G. WORTHING, *University of Pittsburgh.*

Haas and Guth (Indianapolis Meeting, December, 1937) showed that the rate of emission of photons by a black body, regardless of their energy contents, varies as T^3. The question rises, what are the other black body laws which describe the radiation from the same standpoint? For the displacement law, the procedure is much like that involved in the deduction of Wien's law. Starting with an opaque, evacuated, spherical enclosure with perfectly reflecting walls, which is filled with black body radiation, one considers an adiabatic change of volume and obtains the well-known relation,

$$u_\nu \Delta\nu / u_{\nu_0} \Delta\nu_0 = (T/T_0)^4$$

where $\mu_\nu \Delta\nu$ represents the density of radiant energy associated with the range of frequency $\Delta\nu$ in the neighborhood of the frequency ν. An initial or standard state is indicated by subscript zero. Corresponding photon densities or concentrations $C_\nu \Delta\nu$ and $C_{\nu_0}\Delta\nu_0$ are obtained by dividing the radiant energy densities by $h\nu$ and $h\nu_0$. Treatment of the equation obtained by standard procedure yields $C_\lambda = T^4 f(\lambda T)$ which differs only in the power of T from Wien's equation, namely $u_\lambda = T^5 f(\lambda T)$. Analogous to the simpler form of Wien's equation, we have another, $\lambda_m T$ = constant, with the constant in this case equal to $14{,}320\mu K°/3.921$. Analogous to Planck's law, we have, as used by Haas and Guth, $C_\lambda = (8\pi/\lambda^4)[1/(e^{c_2/\lambda T}-1)]$ of which the right-hand member differs from the corresponding Planck form by the factor hc/λ. The relations presented are thought to be of value in connection with photoelectric and photochemical processes.

4. Correlation of Optical Properties and Photoelectric Emission in Thin Films of Alkali Metals.* HERBERT E. IVES AND H. B. BRIGGS, *Bell Telephone Laboratories.*

According to the theory proposed some time ago,[1] the photoelectric emission from continuous thin films of alkali metal deposited on specular surfaces of other metals, such as platinum, is proportional to the energy absorbed by the alkali metal. This absorption is determined by the absorptive properties of the alkali metal, which are obtainable from its optical constants, and the energy density occurring immediately above the supporting metal, which is obtainable from its optical constants. This theory gives an immediate explanation for the characteristic high ratio of emission for the two principal planes of polarization of the incident light, but has needed, for its test at various wave-lengths, data on the optical constants, through the spectrum, of the alkali metals. These are now available[2] and have been used to calculate the expected photoelectric emission. Concurrently a series of measurements have been made with improved apparatus and technique upon potassium, rubidium and cesium (the crucial phenomena predicted for sodium lie outside the utilizable spectrum range). The spectral emission curves obtained from the three metals, for equilibrium films, show spectral maxima of the predicted character. In each case the experimental curves are broader than those predicted for exceedingly thin (monatomic) films and the maxima are shifted toward shorter wave-lengths. Upon assuming thicknesses greater than monatomic for the films, the computed curves both

broaden and shift toward the violet, yielding finally, for certain choices of thickness, a close correspondence with the experimental results.

* The present paper will appear in full in J. Opt. Soc. Am.
[1] Phys. Rev., Sept. 15, 1931, p. 1209.
[2] J. Opt. Soc. Am. **26**, 238 (1936); **27**, 181 and 395 (1937).

5. Dispersion Formulae and Color Correction.* M. HERZBERGER, *Kodak Research Laboratories.*

The measurements of Kingslake on refractive indices in infra-red show that neither Hartmann's formula, nor related formulae, can give a true dispersion formula, but that the old Helmholtz-Ketteler formula

$$n^2 = a + [b/(\lambda^2 - \lambda_0^2)] + c\nu^2$$

is the only satisfactory one. The author tried it on all the glasses in the manufacturers' catalogs, with the result that he could approximate all glasses with an accuracy of a few units in the fifth decimal throughout the visible spectrum. The formulae allow extrapolation into the near infra-red. Study of the color correction in optical systems brought the proposal to abandon Abbe's ν value as characterizing the dispersion qualities of optical glass. The author proposes instead of this the use of a dispersion function which is zero for a fundamental color and which can be characterized by its value for two other colors. The author shows how the well-known formulae for color correction of thin and thick lens combinations simplify, when this dispersion function is introduced which is proportional and not reciprocal to the dispersion of the magnitude investigated.

* The present paper will appear in full in J. Opt. Soc. Am.

6. Some New Physiological and Toxicological Effects of Roentgen Rays.* DAVID I. MACHT, *Pharmacological Research Laboratory, Hynson, Westcott & Dunning, Inc., Baltimore, Maryland.*

The author's interest in roentgen sickness, a not infrequent occurrence in patients undergoing x-ray therapy, was the starting-point of the present investigation. Zoöpharmacological experiments were performed on ten species of animals and phytopharmacological tests were made on various seeds and on seedlings of *Lupinus albus.*[1] Bloods obtained from the different animals while in normal condition and those taken after their exposure to x-rays were studied and compared by special pharmacological methods devised by the author.[2] The most important finding was the discovery that the blood of the various animals undergoing roentgen irradiation contains a toxic substance which usually does not appear until from twelve to twenty-four hours after such treatment and then persists for several days. The amount of toxin present varies with the region treated with x-rays. After roentgen irradiation, smooth muscle organs respond to pharmacodynamic reagents with a slight decrease in contractility. Studies by a modification of the Thunberg method[3] on oxidation and reduction phenomena in freshly excised brain tissue from animals exposed to x-rays, and from controls which were not, reveal that roentgen irradiation effects some impairment of enzymatic activity followed by recovery if dosage is not excessive. Clinical studies on human blood are in progress. Search is also being made for drugs which may counteract the toxicity of x-rays. Plant-physiological experiments were made on the germination of various seeds and on the root growth of *Lupinus albus* seedlings. Growth of *Lupinus albus* seedlings exposed to from 300 to 500R units was accelerated. Larger doses inhibited root growth. Suitable doses of roentgen rays increased percentage of germination of seeds of wheat, oats, tomato and *Lupinus albus* and also accelerated growth of the embryo plants. Larger doses produced inhibitory or toxic effects.

* The present paper will appear in full in J. Opt. Soc. Am.
[1] Macht and Livingston, J. Gen. Physiol. **4**, 573 (1922).
[2] Macht, Arch. Dermat. and Syph. **36**, 1022 (1937).
[3] Macht and Bryan, J. Am. Chem. Soc. **58**, 729 (1936).

7. Intensity and the Visibility of Wires.* SELIG HECHT AND ESTHER U. MINTZ, *Columbia University.*

We have measured with white light the relation between the visual angle subtended by a wire which just becomes visible, and the brightness of the field behind it. The diameter of the field and the length of the wire were about 15°. Over the whole range of brightness the relation shows two sections, one at low illuminations and another at high illuminations expressing rod and cone activities, respectively, as usually found in visual functions. Considered as recording visual acuity, the widths of the wires represent a variation from 0.07 unit of visual acuity at the very lowest intensities to 120 units at the highest. In this form, the relation between visual acuity and intensity resembles those found with more usual test objects like hooks, broken circles or gratings except that ours covers a greater extent. The common range is about 1 to 60 whereas ours covers more nearly 1 to 1500 of which 1 to 100 is the cone range and 1 to 15 is the rod range. The maximum visual acuity corresponds to an angle of 0.5 second, and to a geometrical image 0.04μ wide at the retina, and is thus about 1/65 of the diameter of a central cone. The data may also be considered as measurements of intensity discrimination. The wire or its image decreases the total illumination on a line of retinal elements by an amount ΔI where I is the general illumination of the field. The angle subtended by the width of the wire then measures $\Delta I/I$, and it is possible that the wire is recognized when the value of $\Delta I/I$ becomes critical for the prevailing intensity. The data actually correspond to the intensity discrimination function, as measured by various investigators. It may therefore be that visual acuity measurements are basically determined by the intensity discrimination function, though other factors such as pupil diameter for a grating and the size of retinal elements are undoubtedly involved.

* The present paper will appear in full in J. Opt. Soc. Am.

8. Colorimetry of the Purkinje and Hering Images.* SIDNEY M. NEWHALL, *Johns Hopkins University.*

Purkinje and Hering after-images[1] were induced by 0.2 sec. stimulation of a paramacular retinal area. Colorimetry was accomplished by projecting the after-image to

the viewing plane of a special tristimulus colorimeter[2] and there matching to it the primary image of the colorimetric mixture. The rather exacting technique requires some special training and precision is low relative to usual colorimetry. The difficulty is due in large measure to the short duration of the aptly termed "flash" images under investigation. Blue arcs enduring 0.5 sec. had been matched without much trouble.[3] Chromatic stimuli of several sorts were employed to permit comparison of after-images and complementaries corresponding to different primary images. The retinal illumination from each sort of stimulus was varied through a range sufficient to include the critical region of transformation from Purkinje to Hering image. Trilinear coordinates and derived measures of these images will be discussed.

* The present paper will appear in full in J. Opt. Soc. Am.
[1] D. B. Judd, Am. J. Psychol. **38**, 507 (1927).
[2] S. M. Newhall, Psychol. Mon. **47**, 199 (1936).
[3] S. M. Newhall, J. Opt. Soc. Am. **27**, 165 (1937).

9. Photometric Relationships Between Complementary Colors.* D. L. MacAdam, *Kodak Research Laboratories.*

Two complementary chromaticities must be additively mixed in the proper brightness ratio in order to produce a neutral mixture. This ratio, L, can be computed from any one of the following expressions:

$$L/y+1/y'=(1+L)/y_n,$$
$$L=(y'/y)\cdot(y-y_n)/(y_n-y'),$$
$$L=(y'/y)\cdot(x-x_n)/(x_n-x').$$

The symbols, x, y, and x', y', are the coordinates of the complementary chromaticities in the I.C.I. chromaticity diagram, in which the point x_n, y_n represents the chromaticity of the desired "neutral" or gray mixture. The three points (x, y), (x_n, y_n), (x', y') must be collinear in this order. Neither of the complementaries need be spectrum colors, but the equations may be used to determine the brightness ratios of spectrum complementaries necessary for neutral mixtures. The great brightness ratios resulting from the low luminosity of the blue end of the spectrum are inconvenient for graphical representation, and can be avoided if the energy ratios of complementary spectrum lights necessary for a neutral mixture are computed. Loci can be traced in the I.C.I. chromaticity diagram to give the "moment" of one millilambert of every chromaticity with respect to any assigned neutral stimulus. The brightness ratio of two complementary chromaticities necessary for a neutral mixture is the reciprocal of their moments per millilambert with respect to the neutral. The locus of constant moment (per millilambert), m, is a conic section having the $y=0$ axis as its directrix, the neutral point as its focus and the moment, m, as its eccentricity. This diagram is of immediate use in the plane vector solution of any problem of additive color mixture. The vector solution of the problem encountered in the projection of additive photographs, the determination of the brightness ratios of the three primaries necessary for a neutral screen, will be outlined. The "neutral" chromaticity desired on the screen need not be the neutral with respect to which the moment loci are drawn. The colorimetric purity of any color is the ratio of the moment per millilambert of that color to the moment per millilambert of the spectrum color collinear with the neutral and the sample color.

* The present paper will appear in full in J. Opt. Soc. Am.

10. Effects of Multiple Reflection Between Surfaces of a Light Filter.* Howard J. Eckweiler, *New York University.*

A study of the effects of multiple reflection between the interfaces of a homogeneous light-filter upon its transmission. Instances where these small effects may lead to appreciable error if they are ignored are cited.

* The present paper will appear in full in J. Opt. Soc. Am.

11. A New Luminosity Filter. Kasson S. Gibson, Ray P. Teele and Harry J. Keegan, *National Bureau of Standards.*

In 1919, Ives published a specification[1] for a luminosity (visibility) filter, the principal constituents of which were copper chloride, cobalt ammonium sulphate, and potassium dichromate. The relative spectral transmission of this filter was a close duplicate of the relative luminosity (visibility) curve, except at the short wave-lengths where the absorption of the dichromate was too strong, the transmission of the filter being practically zero below 460 mμ. This defect, however, was of little importance in the use of the filter at that time in physical photometry, since most of the illuminants were then of very low relative-energy content in the blue and violet. With the gas-filled tungsten lamps and the numerous gas-discharge tubes now in use, however, it was necessary to devise a new luminosity filter for use with the thermopile if the resulting physical photometer were to prove adequate for standardization purposes. Since no adjustment of the dichromate in Ives's solution would importantly improve the filter, it was necessary to find a substitute for this component. Some years ago, it had been noticed that a certain piece of Corning G 34 glass, having an "unsharp cut-off," gave a relatively good fit with the standard I.C.I. luminosity (visibility) curve[2] in the blue region. By means of this filter and with the published Davis-Gibson data[3] on copper sulphate and cobalt ammonium sulphate, together with a small amount of potassium dichromate to control the curve in the violet, very good duplication of the I.C.I. luminosity curve has been obtained. This will be illustrated. The principal defect of the filter is, of course, its lack of reproducibility, resulting from use of the yellow glass. A melt of the G 34 glass which does not have the "sharp cut-off" is apparently obtained only accidentally and has hitherto been considered undesirable. It certainly is not reproducible. In an effort to remedy this situation, the National Bureau of Standards has obtained the small remaining stock of this glass from Corning Glass Works and may possibly be able to issue this part of the filter to those who wish to construct a physical photometer.

[1] H. E. Ives, "The Photometric Scale," J. Frank. Inst. **188**, 217 (1919).
[2] K. S. Gibson and E. P. T. Tyndall, Nat. Bur. Stand. Sci. Pap. 475 (1923).
[3] Nat. Bur. Stand. Misc. Pub. No. 114 (1931).

12. Further Study of the Use of Filters and Barrier-Layer Photo-Cells for Tristimulus Colorimetry. RICHARD S. HUNTER, *National Bureau of Standards.*

A paper[1] on the development of equipment and methods for photoelectric tristimulus colorimetry was given at the 1937 fall meeting of the Optical Society. It was shown that the G.E. barrier-layer photo-cell was well suited for this type of colorimetry and a set of filters was described for use with this cell and incandescent source. Since this previous paper, improvements have been made in the amber and blue filters; the best available filters known at the present time being: (a) For the "X" function (with that portion of the function between 380 and 500 mμ made zero): Corning 330 Signal Yellow, 6.7 mm of melt of 11-9-31, with Corning 397 Medium Shade Aklo, 3.0 mm thick. (b) For the "Y" function: Same as described previously. (c) For the "Z" function: Corning 554 Lantern Blue, 5.0 mm of melt of 2-13-35, with Corning Noviol A Pale, 2.5 mm of melt of 8-1-36. It is the purpose of the present paper to discuss the use of data obtained with these source-filter-photo-cell combinations for the description and specification of colorimetric quantities. Attention has been centered on the development of two methods for the treatment of the data: (1) The first method has sought the most satisfactory means for the conversion of experimental tristimulus data to specifications on the standard I.C.I. basis. It is shown that, because of failure to duplicate sufficiently the ideal spectral functions in the source-filter-photo-cell combinations, conversions to the I.C.I. basis are not possible with a high degree of accuracy unless material standards are employed which are similar in spectral characteristics to the samples being measured and which have I.C.I. specifications accurately determined by some other method. The most satisfactory transformation equations for these conversions will be given and the results of a number of transformations reported. (2) The second project has sought a method for using photoelectric tristimulus data in the description and measurement of colorimetric quantities on a uniform-color-tolerance basis. A uniform-chromaticity-scale triangle was sought which would be similar in shape to Judd's[2] triangle and which would have rectangular axes, as do the triangles described by Breckenridge and Schaub[3] and MacAdam.[4] At the 1937 fall meeting of the Optical Society, Judd[5] reported on an empirical equation designed to give uniform color tolerances for surface-color specifications. This equation has been rewritten to include only quantities which may be measured by the photoelectric tristimulus method. Comparisons will be given of a number of color differences measured by Judd's method and by the photoelectric tristimulus method.

[1] R. S. Hunter, J. Opt. Soc. Am. **28**, 51 (1938).
[2] J. Opt. Soc. Am. **25**, 24 (1935).
[3] J. Opt. Soc. Am. **27**, 226 (1937).
[4] J. Opt. Soc. Am. **27**, 294 (1937).
[5] J. Opt. Soc. Am. **28**, 52 (1938).

13. Selected Ordinates for Luminosity Computations with Black Body Illuminants between 2000°K and 4000°K.* D. L. MACADAM, *Kodak Research Laboratories.*

Wave-lengths have been calculated for use in the thirty selected ordinate method for computing the visual reflectance or transmittance of colored materials illuminated with radiation from black bodies at the temperatures: 2000°K, 2200°K, 2360°K, 2600°K, 2848°K, 3000°K, 3200°K, and 4000°K. Each ordinate of any one of these sets is displaced by an almost constant wave-length interval from the corresponding ordinate of any other set. The average value of this displacement is given very accurately (± 0.05 mμ) by the formula:

$$\Delta\lambda = 7.23 \times 10^4 (1/T_1 - 1/T_2) \text{ m}\mu,$$

where the two sets of wave-lengths correspond to black body illuminants having the Kelvin temperatures, T_1 and T_2, between 2000°K and 4000°K. The numerical constant of the above equation is proportional to the value of the constant ($C_2 = 14{,}330$) adopted for use in Planck's formula. Table XVII of the *Handbook of Colorimetry*[1] gives the wave-lengths of the thirty selected ordinates for the I.C.I. illuminant "A" (2848°K) and the I.C.I. "visibility" function, $\bar{y}$. These wave-lengths, increased by the constant given by the following formula, can be used for any black body illuminant having the temperature, T, between 2000°K and 4000°K:

$$\Delta\lambda = (7.23 \times 10^4 / T) - 25.386 \text{ m}\mu.$$

A. C. Hardy[2] has suggested the use of a transparent plate ruled along the wave-lengths of the selected ordinates for 2848°K when placed over a spectrophotometric curve drawn on some standard size graph paper. Such a plate, displaced toward longer wave-lengths by the amount given by the above formula, will serve for the selection of the ordinates for any temperature, T, between 2000°K and 4000°K. The resulting wave-lengths will have an average error of less than 0.5 mμ, and the largest error, occurring at the shortest wave-length, will not be greater than 3.0 mμ. Errors exceeding 1 mμ occur only for the first one and the last two ordinates, and only within a few hundred degrees of the limits of the temperature range. The effects of such errors are quite negligible for the types of spectrophotometric curves commonly encountered. The selected ordinates for the I.C.I. $\bar{x}$ and $\bar{z}$ functions for general colorimetric computations do not conform with sufficient accuracy to any law of the above form to justify the labor of evaluation of the corresponding constants.

* The present paper will appear in full in J. Opt. Soc. Am.
[1] *Handbook of Colorimetry* (Technology Press, Mass. Inst. Technology, Cambridge, Mass., 1936).
[2] A. C. Hardy, "A New Recording Spectrophotometer," J. Opt. Soc. Am. **25**, 311 (1935).

14. Photometry of Colored Light Sources. W. E. FORSYTHE, B. T. BARNES, AND ANN L. SHRIDER, *Lamp Development Laboratory, General Electric Co., Nela Park, Cleveland, O.*

Six different kinds of lamps giving highly colored light—400-watt high intensity mercury arc, 10,000 lumen sodium lamp, 3200 lumen neon lamp and three fluorescent lamps, a pink, a green, and a blue lamp—were selected and a number of measurements made on their light output by twelve experienced observers at the laboratories at Nela Park. The characteristics of the observers' eyes were determined by having each measure the relative transmission of the Ives'[1] yellow and blue solutions which gave the so-called Y/B ratio for the different observers. First, comparison measure-

ments were made of the luminous output of three 400-watt high intensity mercury lamps using the Lummer-Brodhun contrast photometer with colored screens to overcome the color difference and with two field sizes, that is, the regular 8–10° and the 2° field. The difference obtained varied with the observer and with the screen used, but the average results gave about a 4 percent increase for the small field. Next, the light output of a number of high intensity mercury lamps, 10,000 lumen sodium lamps and 1200 lumen green fluorescent lamps, was measured with the direct comparison photometer using a 2° field and a screen to equalize the color difference and with the Ives-Brady[2] flicker photometer. In order to obtain higher intensities with the fluorescent lamp, four lamps were mounted side-by-side and used as a source. The values obtained with the two photometers checked very well, considering the small number of observers and the range of eye sensitivity found.

Finally, the horizontal candlepower of the six types of lamps giving colored light was measured with the flicker photometer in terms of a tungsten lamp standard obtained from the National Bureau of Standards. In order to obtain sufficient intensity for this measurement, two of the neon lamps, four of the green fluorescent lamps and eight each of the other fluorescent lamps were measured together. The spectral intensity of each of these lamps was measured and the candlepower calculated from the spectral intensity by using the standard I.E.S. luminosity factor and good checks obtained, even with these few observers.

[1] Trans. I. E. S. **10**, 203 (1915).
[2] Phys. Rev. **4**, 222 (1914).

15. On the Magnitude of the Error Resulting from Fluorescence in Spectrophotometric Measurements. KASSON S. GIBSON AND HARRY J. KEEGAN, *National Bureau of Standards.*

In spectrophotometers in which the sample is directly irradiated and the reflected energy then spectrally dispersed and measured, the error caused by fluorescence of the sample is small and probably negligible. However, this may not be the case in those spectrophotometers in which the energy is dispersed before any of it is permitted to fall upon the sample. In this latter type of instrument any flourescent energy from the sample[1] reaches the detector undispersed and may produce a large error in the reflection (or transmission) curve.

In connection with the colorimetric specification of a certain phenolformaldehyde resin, the authors had occasion recently to measure the apparent reflectance of the resin relative to MgO. The color of the sample is a weak red, similar to Munsell R 5/4. The true colorimetric values are somewhat uncertain because of non-uniformity but the values of x and y for I.C.I. illuminant C approximate 0.410 and 0.313, respectively. However, the curve obtained on the G.E. recording spectrophotometer (an instrument of the second type, above) gave values of x and y with I.C.I. illuminant C approximating 0.385 and 0.346, respectively (corresponding to a yellow or brown sample instead of red). These erroneous values were caused by the fact that the curve was much too high in the yellow-green. This in turn was traced to the fact that the sample was excited to a strong reddish fluorescence by the incident yellow-green energy, as may readily be demonstrated by irradiating it with the 546 mμ line from a mercury arc and viewing with a hand spectroscope. However, the effect may easily be overlooked because the sample fluoresces but weakly when irradiated with the 365 mμ line of the mercury arc.

The sample was also measured on two visual spectrophotometers of the first type above, but the results are rendered somewhat uncertain by fading of the sample and the insensibility of the eye in the end regions of the visible spectrum.

A remedy for these various errors and uncertainties would apparently be found if a photoelectric instrument could be designed such that the sample could be placed between two spectrometers, the first spectrometer selecting the wave-length band and preventing undue heating of the sample, the second spectrometer dispersing both the stray energy and the fluorescent energy.

[1] A. C. Hardy, "A Recording Photoelectric Color Analyzer," J. Opt. Soc. Am. and Rev. Sci. Inst. **18**, 114–115 (1929); also *Handbook of Colorimetry* (1936); p. 3, footnote 5.

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